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LIPSCHITZ FUNCTION SPACES FOR ARBITRARY METRICS

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The bounded (real or complex valued) functions on a set S are denoted by $l_\infty(S)$ while c_0 and l_∞ denote the usual sequence spaces. For background, notation and definitions concerning Lipschitz spaces, see [3].

The purpose of this note is to announce the following:

THEOREM. *Let* (S, d) *be an infinite metric space (i.e.,* S *has infinitely many points) and suppose that* $\inf_{s \neq t} d(s, t) = 0$. *Then* $\operatorname{Lip}(S, d)$ *contains a subspace isomorphic with* l_∞ *and* $\operatorname{lip}(S, d^\alpha)$, $0 < \alpha < 1$, *contains a complemented subspace isomorphic with* c_0 *(i.e., it is the range of a continuous projection on* $\operatorname{lip}(S, d^\alpha)$).

Under the hypotheses of the theorem, we obtain two corollaries that were previously unknown in general.

COROLLARY 1. $\operatorname{lip}(S, d^\alpha)$ *is not complemented in* $\operatorname{Lip}(S, d^\alpha)$.

COROLLARY 2. $\operatorname{lip}(S, d^\alpha)$ *is not isomorphic to a dual space.*

This also provides a proof of Theorem 2.6 in [3].

REMARKS. 1. Since l_∞ is a P_1-space (see [2, p. 94]) the subspace of $\operatorname{Lip}(S, d)$ isomorphic to l_∞ is complemented.

2. In case $\inf_{s \neq t} d(s, t) > 0$, it is shown in [3, Lemma 2.5] that $\operatorname{Lip}(S, d) = \operatorname{lip}(S, d) = l_\infty(S)$.

3. If $\operatorname{lip}(S, d^\alpha)$ is separable, the subspace isomorphic with c_0 is automatically complemented (see [2, p. 96]). It has been shown by K. deLeeuw and T. M. Jenkins that the dual of $\operatorname{lip}(S, d^\alpha)$, and hence the space itself, is separable when S is compact (see [3, Theorem 4.5]). It is unknown for exactly which metric spaces $\operatorname{lip}(S, d^\alpha)$ [resp. its dual] is separable. Let us only mention that if S is the unit ball of the sequence space l_1 and d is the norm restricted to S, then $\operatorname{lip}(S, d^\alpha)$, $0 < \alpha < 1$, is not separable. Also, see the example at the end of this paper.

It was shown in [1] that if S is an infinite compact subset of Euclidean space and $0 < \alpha < 1$, then $\operatorname{lip}(S, d^\alpha)$ and $\operatorname{Lip}(S, d^\alpha)$ are isomorphic to c_0

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