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HOLOMORPHIC FUNCTIONS ON BALANCED SUBSETS OF A BANACH SPACE

BY RICHARD M. ARON[1]

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1. The study of nuclearly-entire functions between two complex Banach spaces was begun by Nachbin and Gupta in [7] and [8] and has recently been extended by Dineen to entire functions of α-holomorphy type θ in [3] and [4]. In §2, we describe a generalization of Dineen's work to holomorphic functions on balanced open subsets of a Banach space. This generalization is applied in §3 to characterize the space of holomorphic germs of α-holomorphy type θ over a compact, convex balanced set.

Our notation and terminology will follow that given in [3], [4], and [6]. For convenience, we give the most important terms here. U will always denote an open, balanced nonvoid subset of the complex Banach space. $\mathscr{K}(U)$ will denote the family of compact, convex, balanced subsets of U. For simplicity of notation, we will assume that our holomorphic mappings are complex valued; there is no difficulty in extending all results to Banach space valued mappings.

θ will be a holomorphy type, and $(\mathscr{H}_\theta(U), \tau_{\omega\theta})$ will be the space of holomorphic functions on U of holomorphy type θ, endowed with the locally convex topology $\tau_{\omega\theta}$ [6, pp. 34–35, 43]. When θ is the current holomorphy type, we get the space $(\mathscr{H}(U), \tau_\omega)$ of all holomorphic mappings on U, with the ported topology. c_0^+ denotes the set of all nonnegative sequences of real numbers converging to 0. The space $\mathscr{P}_\theta({}^nE)$ of n-homogeneous polynomials of θ-type on E is called intrinsic if the algebraic and topological definition of $\mathscr{P}_\theta({}^nE)$ depends only on the algebraic and topological definition of E. If U is the unit ball for an equivalent norm on E, we denote by $\| \ \|_{\theta,U}$ the norm on $\mathscr{P}_\theta({}^nE)$ corresponding to U. It will be convenient for us to use various equivalent norms on E, whose unit balls will often be expressed in terms of the unit ball B_1 of a fixed norm on E.

An α-holomorphy type θ is a holomorphy type which satisfies the condition that for each $n \in N$, $(\mathscr{P}_\theta({}^nE), \| \ \|_\theta)$ is intrinsic, and if U and V are the unit balls for two equivalent norms on E such that $cU \subseteq V$ for $c \in R^+$, then

$$c^n \|P_n\|_{\theta,U} \leqq \|P_n\|_{\theta,V},$$

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