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REPRESENTATION OF H^p-FUNCTIONS

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ABSTRACT. Let E be a set of positive measure on the unit circle. Let $f \in H^p$ $(1 \leqq p \leqq \infty)$ and g be the restriction of f to E. It is shown that functions g_λ, $\lambda > 0$, can be constructed from g so that $g_\lambda \to f$. We also characterize those functions g on E which are restrictions of functions in H^p $(1 < p \leqq \infty)$.

In the following, the space H^p $(1 \leqq p \leqq \infty)$ will, according to the context, be either the Hardy class of analytic functions in the open unit disc D or the space of the corresponding boundary value functions, viz the subspace of "analytic" functions in $L^p(C)$, C being the unit circle. If $E \subset C$ has positive measure then it is well known (see [3]) that a function in H^p cannot vanish on E without being identically zero. Thus, theoretically at least, $f \in H^p$ is uniquely "determined" by its values on E. In the present work we address ourselves to the problem of recovering functions in H^p from their restrictions to E. Theorem I gives an explicit constructive solution to this problem. The allied problem of characterizing the restrictions to E of functions in H^p $(1 < p \leqq \infty)$ is solved in Theorem II. To the best of our knowledge, the only known results relating to these problems are due to the author [4] where the case $p = 2$ is dealt with.

THEOREM I. *Let $E \subset C$ with $m(E) > 0$. Suppose that $1 \leqq p \leqq \infty$, $f \in H^p$ and that g is the restriction of f to E. For each $\lambda > 0$ define analytic functions h_λ, g_λ on D by*

$$h_\lambda(z) = \exp\left\{-\frac{1}{4\pi}\log(1+\lambda)\int_E \frac{e^{i\theta}+z}{e^{i\theta}-z}\,d\theta\right\}, \qquad z \in D,$$

$$g_\lambda(z) = \lambda h_\lambda(z)\frac{1}{2\pi i}\int_E \frac{\bar{h}_\lambda(w)g(w)\,dw}{w-z}, \qquad z \in D.$$

Then as $\lambda \to \infty$, $g_\lambda \to f$ uniformly on compact subsets of D. Moreover for $1 < p < \infty$ we also have $\|g_\lambda - f\|_p \to 0$ as $\lambda \to \infty$.

THEOREM II. *Let $E \subset C$ with $0 < m(E) < m(C)$. For $g \in L^1(E)$ let g_λ be as in Theorem* I. (a) *If $1 < p < \infty$ then a function $g \in L^p(E)$ is the restriction to E of some $f \in H^p$ if and only if $\sup_{\lambda > 0}\|g_\lambda\|_p < \infty$.* (b) *A function $g \in L^\infty(E)$ is the restriction to E of some $f \in H^\infty$ if and only if $\sup_{p > 1}\limsup_{\lambda\to\infty}\|g_\lambda\|_p < \infty$.*

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