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INDUCED REPRESENTATIONS OF C^*-ALGEBRAS

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In this announcement we will indicate how Mackey's definition [7] of induced representations of locally compact groups can be generalized to the setting of C^*-algebras, and how the imprimitivity theorem [8] can be formulated in this setting. Proofs, as well as discussion of other theorems in the theory of induced representations, will appear elsewhere.

Let G be a locally compact group, and let H be a closed subgroup of G. Let $C_c(G)$ and $C_c(H)$ denote the algebras (under convolution) of continuous complex-valued functions of compact support on G and H respectively, viewed as dense involutory subalgebras of the group C^*-algebras [5] of G and H. Then elements of $C_c(G)$ can be convolved on the right by elements of $C_c(H)$, and under this action $C_c(H)$ acts as an algebra of right centralizers [6] on $C_c(G)$.

Let Δ and δ denote the modular functions of G and H respectively, and let γ be the function on H defined by

$$\gamma(s) = (\Delta(s)/\delta(s))^{1/2}$$

for $s \in H$. Let P denote the linear map from $C_c(G)$ onto $C_c(H)$ defined by

$$P(f)(s) = \gamma(s)f(s)$$

for $f \in C_c(G)$ and $s \in H$. Then P commutes with the involutions. Furthermore, a reformulation of a theorem of Blattner [4] says that P is a positive map, in the sense that $P(f^* * f)$ is a positive element of the pre-C^*-algebra $C_c(H)$ for all $f \in C_c(G)$. Now let the right action of $C_c(H)$ on $C_c(G)$ be redefined by $f \cdot \phi = f * (\gamma\phi)$ for $f \in C_c(G)$ and $\phi \in C_c(H)$, where $\gamma\phi$ denotes the pointwise product of γ and ϕ. Under this new action $C_c(H)$ still acts as an algebra of right centralizers on $C_c(G)$, but now P satisfies the conditional expectation property

$$P(f \cdot \phi) = P(f) * \phi$$

for $f \in C_c(G)$ and $\phi \in C_c(H)$. In general P is not norm-continuous. However, P is relatively bounded, in the sense that for any $g \in C_c(G)$ the map

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