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A NEW EXACT SEQUENCE FOR K_2 AND SOME CONSEQUENCES FOR RINGS OF INTEGERS

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Suppose R is a Dedekind domain with field of fractions F and at most countably many maximal ideals P. Using methods from the theory of algebraic groups, Bass and Tate [B-T] have proved the exactness of the sequence

$$K_2(R) \to K_2(F) \xrightarrow{t} \coprod_P K_1(R/P) \to K_1(R) \to K_1(F) \to \cdots$$

where t is induced by the tame symbols on R. They have also asked whether this sequence remains exact with "$0 \to$" inserted on the left when R is a ring of algebraic integers. In this note we announce an affirmative response when R is a discrete valuation ring, and a proof that the resulting sequence is split exact under certain additional hypotheses on R. In addition, we derive consequences of these results for a ring, $\mathfrak{O}$, of integers in a number field. Among these are

(1) a complete determination of the groups $K_2(\mathfrak{O}/\mathfrak{a})$ for any ideal $\mathfrak{a}$ of $\mathfrak{O}$; and

(2) examples of rings of integers $\mathfrak{O}$ for which $K_2(\mathfrak{O})$ is not generated by symbols and $K_2(2, \mathfrak{O}) \to K_2(3, \mathfrak{O})$ is not surjective. Detailed proofs will appear elsewhere.

1. **The exact sequence.** Let A be a discrete valuation ring with field of fractions K and residue field k. Define the tame symbol [Mi, Lemma 11.4] $t: K_2(K) \to K_1(k) \approx k^*$ by $t(\{u\pi^i, v\pi^j\}) = (-1)^{ij}\bar{u}^j\bar{v}^{-i}$, $u, v \in A^*$, where π generates the maximal ideal of A.

THEOREM 1. *The sequence*

$$0 \to K_2(A) \to K_2(K) \xrightarrow{t} K_1(k) \to 0$$

is exact. Moreover, if A is complete and k is perfect, this sequence is split exact.

The methods used in this proof are elementary in the sense that they

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