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SET VALUED TRANSFORMATIONS

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The present note summarizes some results in a new algebraic topological approach to set valued transformations and initiates a theory of their fixed points applicable to the case that the images are not acyclic. These ideas extend to min max theorems where again a basic generalization is obtained [2]. Our developments are based on the existence of homomorphisms of certain homology groups, in a crucial range only, induced by suitably defined multivalued homotopies (cf. Theorems 1 and 3 below).

Let X and Y be paracompact spaces and suppose $h: X \times I \to Y$ is a set valued uppersemicontinuous (usc) transformation. Let $\Gamma(h)$ be the graph

$$\Gamma(h) = \bigcup \{(x, s, y) | y \in h(x, s)\} \subset X \times I \times Y.$$

Let p_1 be the projection of $\Gamma(h)$ onto X, p_2 the projection onto Y and P_1 the projection of $\Gamma(h)$ onto $X \times I$.

For each $s \in I$ the *singular set* $S^p(s)$ is defined by

$$S^p(s) = \{x | H^r h(x, s) \not\approx 0 \text{ for some } r < p\}$$

where H^* refers to Alexander reduced cohomology over the coefficient field Q and closed support family. Write

$$S^p = \bigcup_{s \in I} S^p(s) .$$

We say p_1 is *almost p solid*, ApS, if for any neighborhood $N(y_0)$ in a suitable neighborhood base at $y_0 \in Y$, there is at most a finite subset of S^p, independent of s, such that $h(x, s) \cap N(y_0) \neq \varnothing$, $x \in S^p$, does not imply $h(x, s) \in N(y_0)$ and $h(x, s)$ is uniformly usc for fixed x.

We write $f \sim_{pq} g$ if h_s, $s \in I$, is acyclic for $p \leqq m \leqq q \leqq \infty$ and $p_1(h)$ is ApS. The basic theorem for our purpose is

THEOREM 1. *If $f \sim_{pq} g$, $q \geqq p + 2$ and h describes the homotopy then $h(m)^*: H^m(Y) \to H^m(X \times I)$ exists for $p + 2 \leqq m \leqq q$ and $f^*(m) = g^*(m)$ for this range of m values.*

If X and Y are compacta, a condition designated by (C) is

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