BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 4, July 1972

AZUMAYA, SEMISIMPLE AND IDEAL ALGEBRAS[1]

BY M. L. RANGA RAO

Communicated by Dock S. Rim, January 31, 1972

Introduction. If A is an algebra over a commutative ring, then for any ideal I of R the image of the canonical map $I \otimes_R A \to A$ is an ideal of A, denoted by IA and called a scalar ideal. The algebra A is called an ideal algebra provided that $I \to IA$ defines a bijection from the ideals of R to the ideals of A. The contraction map defined by $\mathfrak{A} \to \mathfrak{A} \cap R$, being the inverse mapping, each ideal of A is a scalar ideal and every ideal of R is a contraction of an ideal of A. It is known that any Azumaya algebra is an ideal algebra (cf. [**2**], [**4**]).

In this paper we initiate the study of ideal algebras and obtain a new characterization of Azumaya algebras. We call an algebra finitely generated (or projective) if the R-module A is, and show that a finitely generated algebra A is Azumaya iff its enveloping algebra is ideal (1.7).

Hattori introduced in [**9**] the notion of semisimple algebras over a commutative ring as those algebras whose relative global dimension [**10**] is zero. If R is a Noetherian ring, central, finitely generated semisimple algebras have the property that their maximal ideals are all scalar ideals [**6**, Theorem 1.6]. It is also known that for R, a Noetherian integrally closed domain, finitely generated, projective, central semisimple algebras are maximal orders in central simple algebras [**9**, Theorem 4.6]. Therefore by Remark 2.4, over Dedekind domains finitely generated, central, projective semisimple algebras are ideal algebras. (The converse is trivial by [**5**, 8.1].) The Endo-Watanabe example of a semisimple algebra over a discrete valuation ring that is not Azumaya [**7**, Theorem 1] then assures that the class of finitely generated, central ideal algebras is strictly larger than that of Azumaya algebras. Over artinian rings, however, finitely generated central ideal algebras are Azumaya (Corollary 1.10).

Semisimple algebras need not be projective over their centers even if the centers are artinian [**7**] and, hence, not all semisimple algebras are ideal algebras. However, as mentioned above, over artinian rings or over Dedekind domains, finitely generated, central projective semisimple algebras are precisely ideal algebras.

We do not know whether or not, over a Noetherian ring, every finitely

---

AMS 1970 *subject classifications*. Primary 16A16; Secondary 16A18, 16A60.
Key words and phrases. Noetherian rings, integrally closed domains, Dedekind domains, scalar ideals, Azumaya algebras, ideal algebras, separable algebras, discrete valuation ring, relative homological dimension, faithfully flat modules, semisimple algebras, maximal orders.

[1] These results are part of the author's doctoral dissertation written under the direction of Professor Carl Faith at Rutgers, The State University, New Brunswick, New Jersey.