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CURVATURE FUNCTIONS FOR 2-MANIFOLDS WITH NEGATIVE EULER CHARACTERISTIC

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Introduction. In [**4**], [**5**], [**6**] we posed the problem of describing the set of Gaussian curvature functions K that a given 2-manifold M can possess. We observed that if M is compact, the Gauss-Bonnet formula

$$\int_M K\,dA = 2\pi\chi(M)$$

imposes a sign condition on K. For example, K must be negative somewhere if the Euler characteristic $\chi(M)$ is negative. We have asked if these sign conditions are sufficient conditions on a given function $K \in C^\infty(M)$ for K to be the Gaussian curvature of some Riemannian metric on M.

To solve this problem, we have considered the more specific problem of attempting to realize K as the curvature of a metric $\hat{g}$ pointwise conformal to a given metric g, so that $g = e^{2u}g$ for some $u \in C^\infty(M)$. If k is the curvature of the metric g, this becomes the problem of solving the nonlinear elliptic equation [**5**, §5]

$$\Delta u = k - Ke^{2u} \tag{1}$$

on M, where Δ is the Laplacian in the g metric.

In our previous studies ([**4**], [**5**], [**6**]) we have given a complete discussion of these questions if $\chi(M) = 0$ as well as some partial results for $\chi(M) \neq 0$. We were also able to obtain information concerning the curvature functions of some open 2-manifolds. In this case there is no Gauss-Bonnet restriction, so one expects that *any* smooth function is the curvature of some Riemannian metric. A survey of related literature is given in [**4**], [**5**], [**6**] to which we refer the reader.

In this paper we completely settle the description of the set of Gaussian curvature functions on compact M with $\chi(M) < 0$ as well as the open manifold case for all open 2-manifolds obtained as "nicely punctured" compact manifolds. Thus this question remains open only in the case of S^2 for which partial results have been obtained in [**3**], [**5**], [**11**].

It turns out that the differential equation (1) becomes easier to work with if one frees it from the geometric situation and instead considers the more general equation $\Delta u = f + he^{2u}$ where f and h are prescribed functions, not tied to geometric considerations. In this paper we are concerned with

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