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ON THE REALIZABILITY OF MODULES OVER THE STEENROD ALGEBRA

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1. **Introduction.** Let A denote the mod 2 Steenrod algebra. There is good reason to believe that knowledge both of the structure of the category of A-modules and of the subcategory of those A-modules which are the cohomology of spectra (i.e. are realizable), will give us great insight into the structure of the stable homotopy category of spaces. So far the structure and realizability problems have received little attention of a general nature, work having mostly centered around the specific insights needed to attack specific problems. For example, with respect to the realizability problem, research to date gives us little more than the realizability of specific A-modules (e.g. [**2**], [**3**] and [**4**]).

This paper describes a step in the attack of these problems from a more general point of view.

I begin by recalling work by J. F. Adams and myself [**1**] concerning the structure problem. Using this work I describe a family of natural constructions analogous to the killing of homotopy groups. These constructions can be performed both on A-modules and on spectra, and under mod 2 cohomology they correspond. Therefore if an A-module is realizable then so are the modules obtained from it by the constructions.

A more detailed exposition of this work and its ramifications is in preparation.

2. **Modules over the Steenrod algebra.** The structure of the mod 2 Steenrod algebra, A, is well known and we are in particular interested in a family of elements best describable in terms of the Milnor basis [**5**]: Let P_t^s be the dual of $\xi_t^{2^s}$. Then an easy computation using the product formula gives us that $(P_t^s)^2 = 0$ if $s < t$. This allows the definition:

DEFINITION 2.1. Let M be a left A-module; if $s < t$ then define $H(M, P_t^s) = \ker P_t^s | M/P_t^s M$, *the P_t^s-homology group of* M. This is clearly a functorial invariant of M. It is also graded by the grading on M. In what follows we will only consider P_t^s with $s < t$.

These are strong invariants as evidenced by the following:

THEOREM 2.2 [**1**]. *If $f: M \to N$ is a map of bounded below (b.b.) A-modules and $H(f, P_t^s)$ is an isomorphism for all $s < t$ then M and N are stably iso-*

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