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REMARKS ON SOME RESULTS OF GELFAND AND FUKS

BY VICTOR W. GUILLEMIN[1]

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By a *topological* Lie algebra over $\mathbf{R}$ we will mean a Lie algebra, $\mathscr{L}$, whose underlying vector space has a topology for which the bracket operation: $\mathscr{L} \times \mathscr{L} \to \mathscr{L}$ is continuous. One can associate with such a Lie algebra a complex, the i-cochains of which are all continuous alternating i-linear maps:

$$(*) \qquad \omega : \mathscr{L} \times \cdots \times \mathscr{L} \to \mathbf{R}$$

and the coboundary operator defined by:

$$(**) \qquad d\omega(\zeta_1, \ldots, \zeta_{i+1}) = \sum (-1)^{j+k} \omega([\zeta_j, \zeta_k], \zeta_1, \ldots, \hat{\zeta}_i, \ldots, \hat{\zeta}_j, \ldots, \zeta_{i+1})$$

the cohomology of this complex will be denoted $H(\mathscr{L}, \mathbf{R})$. Gelfand and Fuks have proved the following remarkable result.

THEOREM. *Let X be a smooth compact oriented manifold. Let $\mathscr{L}$ be the Lie algebra of smooth vector fields on X topologized by its C^∞ topology. Then $H(\mathscr{L}, \mathbf{R})$ is finite dimensional in all dimensions.*

See [**1**].

Figuring in their computations is a certain subcomplex of $(*)$ which they call the *diagonal* complex. It consists of all i-cochains $(*)$ having the property

$$\omega(\zeta_1, \ldots, \zeta_i) = 0 \quad \text{when } \operatorname{supp} \zeta_1 \cap \ldots \cap \operatorname{supp} \zeta_i = \Phi.$$

The cohomology of this diagonal complex they denote by $H_\Delta(\mathscr{L}, \mathbf{R})$. To describe their result about $H_\Delta(\mathscr{L}, \mathbf{R})$, consider the formal power series ring $\mathbf{R}[[x_1, \ldots, x_n]]$ generated by the n indeterminates $x_1, \ldots, x_n$. The $\mathbf{R}$-linear derivations of this ring are a Lie algebra over $\mathbf{R}$ which we will denote by L. The $\mathscr{M}$-adic topology on the formal power series ring induces a topology on L. Let $H(L, \mathbf{R})$ be the cohomology of L with respect to this topology. The result of Gelfand-Fuks is:

THEOREM. *There is a spectral sequence whose E^2 term is the tensor product $H(X, \mathbf{R}) \otimes H(L, \mathbf{R})$ and whose E^∞ term is $H^j(X, \mathbf{R})$ for $j \leqq n$, and $H_\Delta^{j-n}(\mathscr{L}, \mathbf{R})$ for $j \geqq n$.*

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