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HOMOTOPY GROUPS OF FINITE *H*-SPACES

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In this announcement we present results about the homotopy groups of H-spaces having the homotopy type of finite CW-complexes. We call such spaces *finite H-spaces*. We always assume our spaces are connected. In the sequel we always use X to denote a finite H-space. In some statements we refer to a direct sum of cyclic groups. We do not rule out the case that the sum is zero.

Let $\tilde{X}$ be the fibre of the canonical map

$$X \to K(\Pi_1(X), 1).$$

It is well known that this "universal covering space" $\tilde{X}$ is a finite H-space.

THEOREM 1. *$\Pi_4(X)$ is a direct sum of groups of order* 2, $\dim \Pi_4(X)' = \dim \ker \mathrm{Sq}^2 : H^3(\tilde{X}:Z_2) \to H^5(\tilde{X}:Z_2)$.

PROOF. Since $\tilde{X}$ is a finite H-space, it suffices to work with simply connected X. We use the exact sequence of J. H. C. Whitehead,

$$\to H_{n+1}(X:Z) \xrightarrow{\nu_n} \Gamma_n(X) \xrightarrow{\lambda_n} \Pi_n(X) \xrightarrow{h_n} H_n(X:Z) \to .$$

Results of Browder [3] and Hilton [7] give $\Gamma_4(X) \cong H_3(X:Z_2)$. Browder's Theorem 6.1 of [3] yields

LEMMA 2. *Let X be simply connected, then $H_4(X:Z) = 0$.*

From [7] we obtain ν_4 as the composite

$$H_5(X:Z) \xrightarrow{r} H_5(X:Z_2) \xrightarrow{\mathrm{Sq}^2_*} H_3(X:Z_2)$$

where r is reduction mod 2. The theorem follows.

We remark that if X is simply connected and $H_*(\Omega X:Z)$ torsion free, then Theorem 1 is contained in Bott-Samelson [**2**].

For the remainder of this paper we assume that *X is simply connected and $H_*(\Omega X:Z)$ is torsion free.* We identify $\Gamma_4(X)$, $H_3(X:Z_2)$ and $\Pi_3(X) \otimes Z_2$, and continue to use ν_4. For $k \geqq 3$, $\eta_k : S^{k+1} \to S^k$ is the essential map.

THEOREM 3. *The following sequence is exact,*

$$0 \to \Pi_4(X) \xrightarrow{\eta_4^*} \Pi_5(X) \xrightarrow{h_5} H_5(X:Z) \xrightarrow{\nu_4} \Pi_3(X) \otimes Z_2 \xrightarrow{\eta_3^*} \Pi_4(X) \to 0,$$

with $\ker h_5 = \operatorname{tors} \Pi_5(X)$, *the torsion subgroup of* $\Pi_5(X)$.

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