BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 4, July 1972

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NONCOBORDANT FOLIATIONS OF S^3

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Communicated by Emery Thomas, December 15, 1971

In this note, we will sketch the construction of uncountably many noncobordant foliations of S^3, and a surjective homomorphism $\pi_3(B\Gamma_1^r) \twoheadrightarrow R$ $[2 \leqq r \leqq \infty]$, where $B\Gamma_1^r$ is the classifying space for singular C^r codimension one foliations constructed by Haefliger ([3], [4]).

Godbillon and Vey have recently discovered certain cohomology classes associated with foliations, and more generally, with Γ_p^r-structures or singular C^r codimension p foliations [2]. The cohomology invariant Γ_F is defined very simply for a codimension one, transversely oriented foliation F determined by a C^2 one-form ω. The condition of integrability for a one-form is $d\omega \wedge \omega = 0$. Then for some one-form θ, $d\omega = -\theta \wedge \omega$. Γ_F is defined to be the deRham cohomology class of the closed form $\theta \wedge d\theta$. If F is not transversely oriented, Γ_F may be defined via two-sheeted covers. If F is C^2 but not given by a C^2 one-form, it is still possible to define Γ_F. Γ_F depends only on F, not on ω and θ, and is natural; so if $f: M \to N$, where N has foliation F and f induces a foliation f^*F on M, then $\Gamma_{f^*F} = f^*\Gamma_F$. It follows that $\Gamma_F[M^3]$ is an invariant of the cobordism class of F. That is, if $\partial N^4 = M_1 + -M_2$, and if N^4 has foliation F transverse to ∂N^4 inducing F_1 on M_1 and F_2 on M_2, then $\Gamma_{F_1}[M_1] = \Gamma_{F_2}[M_2]$.

The form $\theta \wedge d\theta$ may be interpreted as a measure of the helical wobble of the leaves of F, as in Figure 1. In order that the cohomology class Γ_F be nontrivial, there must be some kind of global phenomenon corresponding to helical wobble.

Now consider the hyperbolic plane H^2 and its unit tangent bundle $T_1(H^2)$. There is a foliation F of $T_1(H^2)$ invariant under the isometries of H^2: each leaf of F consists of the forward unit tangents to a family of parallel geodesics. In non-Euclidean geometry, parallel means asymptotic

AMS 1969 *subject classifications*. Primary 5136.

[1] The author was supported as a reserach assistant under NSF grant GP 29073.