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BOOK REVIEWS

Hyperbolic Manifolds and Holomorphic Mappings by Shoshichi Kobayashi. 148 pp. Marcel Dekker, 1970. $11.75.

The theory of analytic functions of a complex variable is one of the most beautiful and pervasive subjects in mathematics. It has a strongly geometric aspect, which may roughly be labelled as the study of holomorphic mappings. One of the most important tools for this study is the Schwarz-Pick-Ahlfors lemma concerning the influence of curvature on such holomorphic mappings. The theme of this monograph is that the aforementioned lemma, together with elementary reasoning of a point-set topological character, leads to an interesting unified study of some properties of holomorphic mappings. The book is extremely well written, essentially selfcontained, and is recommended to beginning graduate students in mathematics as well as to specialists in complex variables. The reviewer's main criticism is that the deeper aspects of the use of differential-geometric methods for the study of holomorphic mappings, such as the Nevanlinna theory or the applications to algebraic geometry, go essentially unmentioned, so that a nonknowledgeable reader is left unaware of the enormous further possibilities of the field. I shall briefly discuss one part of the philosophy behind the methods presented in this book, shall then summarize some of the main results, and shall finally offer a few comments.

(1) A fundamental property of holomorphic functions is the Schwarz-Pick lemma, which states that a holomorphic mapping $w = f(z)$ from the unit disc $D = \{z : |z| < 1\}$ in the complex plane to itself is *distance decreasing* for the Poincaré metric $ds_D^2 = dz\, d\bar{z}/(1 - |z|^2)^2$. This result has been considerably generalized. To begin with, Ahlfors gave the following theorem: Let S be a Riemann surface having a conformal metric ds_S^2 whose Gaussian curvature K_S is everywhere $\leqq -4$ (recall that the Poincaré metric has constant negative curvature $K_D = -4$). Then a holomorphic mapping $f : D \to S$ is distance decreasing relative to the metrics ds_D^2 and ds_S^2. Because of the elementary fact that holomorphic sectional curvatures decrease on complex submanifolds, the Ahlfors theorem remains valid for a holomorphic mapping $f : D \to M$ where M is a complex manifold having an Hermitian metric ds_M^2 whose holomorphic sectional curvatures are everywhere negative and bounded away from zero. Finally the same distance decreasing result holds when D is an arbitrary bounded symmetric domain in C^n and ds_D^2 is the (suitably normalized) canonical invariant metric.

Continuing in a similar vein, it was proved by Chern and Kobayashi