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STRONGLY ADDITIVE TRANSFORMATIONS AND INTEGRAL REPRESENTATIONS WITH MEASURES OF NONLINEAR OPERATORS

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In the development of the representation theory for linear transformations on function spaces by integrals, historically three different stages can be distinguished: in stage (I) one considers scalar valued transformations (functionals) on scalar valued functions, in stage (II) the transformations (stage (IIA)) or the functions (stage (IIB)) assume values in a topological vector space, and in stage (III) one studies the case of (vector valued) transformations on a space of vector valued functions. If we restrict ourselves to Banach spaces, as for the space of continuous functions on a compact Hausdorff space, stage (I) is marked by the theorems of F. Riesz [**19**] and S. Kakutani [**14**], (IIA) by the results of R. G. Bartle, N. Dunford and J. Schwartz [**1**], (IIB) by the investigations of S. Bochner and A. E. Taylor [**6**] and I. Singer [**20**], (III) by the results of [**5**], of N. Dinculeanu [**8**], and C. Foiaş and I. Singer [**10**]. In a similar way, the history of representation theory for linear maps on L^p-spaces could be retraced. Recently however, most interesting features have been added to the theory by the introduction and consideration of a wide class of nonlinear transformations, for example those T, for which $T(f_1 + f_2) = Tf_1 + Tf_2$ for all pairs of functions f_1, f_2 for which the intersection of the sets $\{t : f_i(t) \neq 0\}$, $i = 1, 2$, is empty or has measure zero. The theory of these "additive" transformations on the L^p-spaces was begun with results of L. Drewnowski and W. Orlicz [**9**] and V. Mizel and K. Sundaresan [**15**], [**16**], [**17**] (stage (I)) and rather quickly reached the stages (II) and (III) by a subsequent paper of V. Mizel and K. Sundaresan [**18**]. As for the nonlinear transformations on spaces of continuous functions it seems evident that the additivity alone is not sufficient to build up a satisfactory representation theory. Stage (I) of their theory began with the research of R. V. Chacon and N. A. Friedman [**7**] and of N. A. Friedman and M. Katz [**11**], [**12**] and with their introduction of a generalized additivity property (called "strong additivity" below). Since these papers appeared the writer of this note worked out the theory in its stage (III) following the line of his earlier work in linear theory and presented his results at a conference on vector measures at the University of Utah in April 1971 not being aware of the fact that the theory had

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