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GENERALIZED PRODUCT THEOREMS FOR TORSION INVARIANTS WITH APPLICATIONS TO FLAT BUNDLES

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This note announces generalizations of the product theorems for Wall invariants and Whitehead torsions due to Gersten [5], Siebenmann [7, Chapter VII], and Kwun and Szczarba [6], and applies these theorems to study torsion invariants of the total space of a flat bundle. The generalized product theorems are described in §§1 and 2. The applications are found in §3.

These theorems were discovered in an attempt to understand more clearly the orientation phenomena discovered in [1] and [2] by concentrating attention on bundles in which "orientation" is a complete bundle invariant. The author would like to thank D. Sullivan whose use of the word "flat" in a private conversation stimulated this work.

0. **Basic algebraic definitions and notations.** Let R be a commutative ring with unit. (Usually $R = Z$, the ring of integers, or Q the rational numbers.) For any group π, $\mathfrak{P}R(\pi)$, and $\sum\mathfrak{P}R(\pi)$ will denote the category of finitely generated projective modules over $R(\pi)$, and the category with objects (P, f) with $P \in \mathfrak{P}R(\pi)$ and $f: P \to P$ and $R(\pi)$ isomorphism. A morphism $g:(P_1, f_1) \to (P_2, f_2)$ is an $R(\pi)$ homomorphism $g: P_1 \to P_2$ such that $f_2 g = g f_1$.

$K_0R(\pi)$ and $K_1R(\pi)$ will be usual algebraic K-theoretic groups (cf. [3, pp. 344–348]). $[P]$ or $[P, f]$ will denote the class of P and (P, f) in $K_0R\pi$ and $K_1R\pi$ respectively. The quotient of $K_1R(\pi)$ by the subgroup $\pm\pi$ will be denoted Wh $R(\pi)$ and will be called the R-Whitehead group of π. When $R = Z$ this is the usual Whitehead group. If $j:\pi \to \pi'$ is a homomorphism, j_* will denote any of the induced maps on K_0, K_1, or Wh.

Let A and B be groups and $\alpha: B \to \operatorname{Aut} A$ be a homomorphism. Then $A \times_\alpha B$ will denote the semidirect product of A and B with respect to α. As sets $A \times_\alpha B = A \times B$. The multiplication on $A \times_\alpha B$ is given by $(a, b)(a', b') = (a\alpha(b)(a'), bb')$. The functions $k: A \to A \times_\alpha B$, $p: A \times_\alpha B \to B$, and $s: B \to A \times_\alpha B$ given by $k(a) = (a, 1)$, $p(a, b) = b$, and $s(b) = (1, b)$ are homomorphisms. α extends to a homomorphism, also denoted by $\alpha, \alpha: B \to \operatorname{Aut} R(A)$.

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