BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 3, May 1972

HARMONICS ON STIEFEL MANIFOLDS AND GENERALIZED HANKEL TRANSFORMS

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Communicated by W. Fuchs, November 4, 1971

Introduction and notation. The following two theorems are basic to the classical theory of spherical harmonics and their importance in analysis is well known.

THEOREM 1. (CARTAN [**2**].) *Suppose p is a function on the* $(n - 1)$-*sphere* S^{n-1} *which transforms under* $SO(n)$ *according to an irreducible representation of highest weight* $(k, 0, \ldots, 0)$. *Then p extends to a harmonic polynomial on* $\mathbf{R}^n$ *satisfying the homogeneity condition* $p(rX) = r^k p(X)$ *for all* $r > 0$ *and* $X \in \mathbf{R}^n$.

THEOREM 2. (BOCHNER [**1**].) *Suppose f is a radial function on* $\mathbf{R}^n$, *p is as in Theorem* 1, *and* $F = fp$ *is square-integrable. Then the Fourier transform of F is gp where g is the Hankel transform of f of order* $k + ((n - 2)/2)$.

In this note we announce an extension of these theorems to the setting of Stiefel manifolds and matrix space. Our work makes it possible to construct holomorphic discrete series representations for the real symplectic group by decomposing a tensor product of certain projective representations introduced earlier by Shale and Weil. (See Weil [**11**] and also Shalika [**10**].) Proofs of the results announced here and their application to the construction of discrete series will appear elsewhere.

We let $M_{n,m}$ denote the $n \times m$ real matrix space, $S^{n,m}$ the Stiefel manifold of matrices $V \in M_{n,m}$ such that ${}^tVV = I_m$, and P_m the cone of $m \times m$ positive-definite symmetric matrices. The rotation group $SO(n)$ acts on $S^{n,m}$ and $M_{n,m}$ by left matrix multiplication so that $S^{n,m} \cong SO(n)/SO(n-m)$. Corresponding to the decomposition $M_{n,m} = S^{n,m} \times P_m$ we have the integral formula

$$\int_{M_{n,m}} F(X)\,dX = c_{n,m} \int_{P_m} \left(\int_{S^{n,m}} F(VR^{1/2})\,dV \right) (\det R)^{\nu}\,dR$$

where $\nu = (n - m - 1)/2$, $c_{n,m}$ is a constant depending only on n and m, and dV is $SO(n)$-invariant. The algebra $M_{m,m}$ acts on $M_{n,m}$ by right matrix multiplication.

AMS 1969 *subject classifications*. Primary 2257, 4216; Secondary 2265, 3327.
Key words and phrases. Generalized spherical harmonics, Stiefel manifold, matrix space, class m representations of $SO(n)$, holomorphic representations of the general linear group, Fourier transforms on matrix space, generalized Hankel transforms, Bessel functions of matrix argument.