BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 3, May 1972

DE RHAM'S INTEGRALS AND LEFSCHETZ FIXED POINT FORMULA FOR d'' COHOMOLOGY

BY YUE LIN L. TONG[1]

Communicated by Raoul Bott, October 22, 1971

We give here a brief sketch of a different approach to the Atiyah-Bott type Lefschetz fixed point formula for Dolbeault complexes. Our method is based on an extension to the complex case of de Rham's integral formulas for Kronecker indices [7]. This approach yields results for general fixed point sets, and in particular we shall give here a formula for isolated degenerate fixed points. Details and related results will appear elsewhere.

Following notations in [**1**], [**2**], [**3**], let X be a compact complex analytic manifold of complex dimension n,

$$\Gamma(\Lambda^{p,*}X):0 \to \Gamma(\Lambda^{p,0}) \xrightarrow{d''} \Gamma(\Lambda^{p,1}) \to \cdots \xrightarrow{d''} \Gamma(\Lambda^{p,n}) \to 0,$$

$0 \leqq p \leqq n$, the pth Dolbeault complex, $f:X \to X$ a complex analytic mapping with isolated fixed points, and

$$T_{p,q} = \Lambda^p(d'f^*) \otimes \Lambda^q(d''f^*) \circ f^*:\Gamma(\Lambda^{p,q}) \to \Gamma(\Lambda^{p,q})$$

the induced endomorphisms on the complex. In terms of $T_{p,q}$ we define, as in [**3**],

$$\operatorname{graph}\{T_{p,q}\} \in \Gamma'(\Lambda^{p,q} \boxtimes (\Lambda^{p,q})')$$

where $(\Lambda^{p,*})'$ denotes the geometric dual and Γ' the space of distributions. It is then seen that

$$\operatorname{graph}\{T_p\} = \sum_{q=0}^{n} \operatorname{graph}\{T_{p,q}\} \in H'(\Lambda^{p,*} \boxtimes (\Lambda^{p,*})').$$

Similarly define

$$\Delta_p = \sum_{q=0}^{n} \operatorname{graph}\{I_{p,q}\} \in H'((\Lambda^{p,*})' \boxtimes \Lambda^{p,*})$$

where $I_{p,q}:\Gamma((\Lambda^{p,q})') \to \Gamma((\Lambda^{p,q})')$ is the identity. Analogous to [**3**], [**6**], one deduces from Poincaré duality and Künneth formula that the Lefschetz number

$$L(f^{p,*}) = \sum(-1)^q \operatorname{trace}\{T^*_{p,q}\}$$

AMS 1970 *subject classifications*. Primary 32A25, 53C65, 58G10; Secondary 31B10.
Key words and phrases. Atiyah-Bott formula, isolated degenerate fixed points, Lefschetz number, Laplace operator, Green's form, Bochner's formula, Grothendieck's residue symbol.
[1] This work was supported in part by NSF GP-8839.