BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 3, May 1972

CLASSIFICATION OF THE COMPLETELY PRIMARY TOTALLY RAMIFIED ORDERS WITH A FINITE NUMBER OF NONISOMORPHIC INDECOMPOSABLE LATTICES

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Communicated by Hyman Bass, November 15, 1971

Let K be the $\mathfrak{p}$-adic completion of an algebraic number field and denote by R its ring of integers. We assume that Λ is an R-order in the semisimple finite dimensional K-algebra A. One of the main problems in the representation theory of orders is the classification of those orders Λ which have only a finite number of nonisomorphic indecomposable left Λ-lattices the so-called "orders of finite lattice type." The commutative case has been settled independently by Drozd-Roiter [2] and Jacobinski [3]. For the general case only partial results are known [1], [4], [5], [7].

By Morita equivalence we may assume that $\Lambda/J(\Lambda)$, where $J(\Lambda)$ denotes the Jacobson-radical of Λ, is a finite direct sum of extension fields $\mathfrak{R}_i$ of $\mathfrak{R}$, the residue field of R, say

$$\Lambda/J(\Lambda) \overset{\text{ring}}{\cong} \bigoplus_{i=1}^{m} \mathfrak{R}_i.$$

We choose a finite unramified extension K' of K with ring of integers R' such that the residue field $\mathfrak{R}'$ of R' is a splitting field for the minimum polynomial of $\mathfrak{R}_i$ over $\mathfrak{R}$. Putting $\Lambda' = R' \otimes_R \Lambda$ we have

$$\Lambda'/J(\Lambda') \cong \bigoplus_{i=1}^{n} \mathfrak{R}'.$$

Jacobinski [3, Proposition 1] has shown that Λ is of finite lattice type if and only if Λ' is of finite lattice type. Therefore we may assume that

$$(1) \qquad \Lambda/J(\Lambda) \cong \bigoplus_{i=1}^{n} \mathfrak{R},$$

where $\mathfrak{R}$ is the residue field of R.

We shall classify here those orders of finite lattice type for which $n = 1$ —these are called "completely primary totally ramified" (notation CPTR). By $n(\Lambda)$ we denote the number of nonisomorphic indecomposable left Λ-lattices.

AMS 1970 *subject classifications*. Primary 16A18, 16A48, 16A64.
Key words and phrases. Orders, finite lattice type, completely primary orders.