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THE EXISTENCE OF SOLUTIONS TO CLASSICAL VARIATIONAL PROBLEMS WITHOUT ASSUMING CONVEXITY

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The following is a summary of a paper to appear shortly. It is a part of a Ph.D Thesis submitted to the Senate of the Hebrew University of Jerusalem. It was carried out under the supervision of Professor E. Shamir.

1. We shall deal with the problem of the existence of a solution to the classical variational problem, in parametric form.

2. Let D be a compact region in the x-y plane. (In the following a "compact region" will mean a compact set, such that each two of its points could be joined by a polygon lying entirely (with possible exception of its end-points) in the interior of the set.

3. Let $f(x, y, p, q)$ be defined and have continuous partial derivatives to the second order for every (x, y) in D and every p, q. Also let $f(x, y, p, q)$ be positive homogeneous of the first order in p, q and assume that

$$p^2 + q^2 \neq 0 \Rightarrow f(x, y, p, q) > 0.$$

4. Let P, Q be given points in the x-y plane, and denote by S the family of all curves $(x(t), y(t))$ in D which join P and Q, and such that $x(t)$ and $y(t)$ are absolutely-continuous and satisfy $\dot{x}^2 + \dot{y}^2 \neq 0$ almost everywhere.

5. For each $(x, y) \in D$ we denote by $K(x, y)$ the cone in p, q, f space consisting of the points (p, q, f) for which $f(x, y, p, q) = f$.

6. We shall denote by $\mathrm{In}(x, y)$ the "Indicatrix," that is the set of points in the p-q plane for which $f(x, y, p, q) \equiv 1$.

7. We denote also by $\mathrm{In}^*(x, y)$ the convex-hull of $\mathrm{In}(x, y)$ (in the p-q plane) and we define

$$\mathrm{In}^{**}(x, y) \equiv_{\mathrm{def}} \mathrm{In}(x, y) \cap \mathrm{In}^*(x, y).$$

8. We shall denote by $H(x, y)$ the following set:

$$H(x, y) \equiv_{\mathrm{def}} \left(A \,\middle|\, \sin A = \frac{q}{(p^2 + q^2)^{1/2}};\ \cos A = \frac{p}{(p^2 + q^2)^{1/2}};\ (p, q) \in \mathrm{In}^{**}(x, y) \right).$$

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