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HOLOMORPHIC MAPPINGS: SURVEY OF SOME RESULTS AND DISCUSSION OF OPEN PROBLEMS[1]

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1. **Introduction.** Our purpose is to survey some aspects of the global theory of holomorphic mappings, isolating along the way a few outstanding questions.

The general problem is this: Let M and N be complex manifolds and $f: M \to N$ a holomorphic mapping. Study the position of the image $f(M)$ in N. In particular, if $S \subset N$ is a complex analytic subvariety, then setting $S_f = f^{-1}(S)$, study the $S_f \subset M$ as S varies among the subvarieties of N.

The most important special case of this problem is when $M = \boldsymbol{C}^m$ and $N = \boldsymbol{P}^n$, the complex projective space. Then f may be given by n entire meromorphic functions

$$f(z) = (f_1(z), \ldots, f_n(z)), \qquad z = (z_1, \ldots, z_m) \in \boldsymbol{C}^m.$$

The subvarieties S will be the zero sets of polynomials $p_\alpha(w)$ ($w = (w_1, \ldots, w_n) \in \boldsymbol{C}^n$), and so our question amounts to globally studying solutions of the equations

$$(1) \qquad \begin{aligned} w_j &= f_j(z) \qquad (z \in \boldsymbol{C}^m), \\ p_\alpha(w) &= 0. \end{aligned}$$

The following two examples illustrate the extremes in what is understood about this problem.

EXAMPLE 1. Suppose that $f: \boldsymbol{C} \to \boldsymbol{P}^1$ is an entire meromorphic function, so that (1) reduces to studying the roots of the equation

$$(2) \qquad f(z) = a, \qquad z \in \boldsymbol{C} \text{ and } a \in \boldsymbol{P}^1.$$

The most immediate global property is the *Liouville theorem*, which says that the image $f(\boldsymbol{C})$ is dense in $\boldsymbol{P}^1$, unless of course f is constant. A much more precise result is the *Picard theorem*, which states that a nonconstant f can omit at most two values. Finally, the most penetrating study of the equation (2) is that by *R. Nevanlinna* [**2**], who found that, with at most two exceptional values $a \in \boldsymbol{P}^1$, the "density" of the solutions of (2) in the disc $|z| < r$ is positive as $r \to \infty$. This result is a beautiful and far reaching quantitative refinement of the Picard theorem, and will be discussed further below.

EXAMPLE 2. *Fatou and Bieberbach* found a holomorphic mapping $f: \boldsymbol{C}^2 \to \boldsymbol{P}^2$ which is one-to-one and whose image omits an open set in $\boldsymbol{P}^2$.

[1] Talk presented at the dedication of "Surge L," Stonybrook, New York, October 16, 1971. Printed by invitation of the editors; received October 28, 1971.

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