BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 2, March 1972

PERTURBATION OF EMBEDDED EIGENVALUES[1]

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Communicated by Peter Lax, August 5, 1971

In [**4**] a Weinstein-Aronszajn multiplicity theory for embedded eigenvalues arising from a certain type of "resonance" was developed. The results announced here continue the work of [**4**], and generalize results of [**2**] and [**3**] to embedded eigenvalues of arbitrary finite multiplicity m, and to perturbations of infinite rank. In particular, we are able to discuss certain operators of quantum mechanics. A notable feature of the case $m > 1$ is the appearance of Puiseux series for the resonances, in analogy to their appearance in the perturbation theory of *isolated* eigenvalues of *nonselfadjoint* operators [**6**, Chapters 2 and 7].

1. **Puiseux series for resonances.** Let T be a selfadjoint operator on a separable Hilbert space $\mathscr{H}$, with resolvent $G(z) = (T - z)^{-1}$, and let λ_0 be a point eigenvalue of T of finite multiplicity m. Denote by P the orthogonal projection on $\ker(T - \lambda_0)$. Let A and B be bounded commuting selfadjoint operators on $\mathscr{H}$, and define

$$H(\kappa) = T + \kappa AB.$$

For real κ, $H(\kappa)$ is selfadjoint and we define $R(z, \kappa) = (H(\kappa) - z)^{-1}$. Let Ω be a neighborhood λ_0 in the complex plane, and assume that the operator $Q(z) = AG(z)B$ is bounded and has meromorphic continuations $Q^{\pm}(z)$ from $\Omega^{\pm} = \{z \in \Omega\colon \operatorname{Im} z > 0\}$ to Ω. There is then a simple pole of $Q^+(z)$ at λ_0 with residue APB. The functions $Q^+(z)$ and $Q^-(z)$ will not agree on Ω if the eigenvalue λ_0 is embedded in the continuous spectrum of T. The operator

$$Q_1(z, \kappa) = AR(z, \kappa)B$$

also has meromorphic continuations $Q_1^{\pm}(z, \kappa)$ given by

$$I - \kappa Q_1(z, \kappa) = [I + \kappa Q(z, \kappa)]^{-1}.$$

It is the poles of $Q_1^+(z, \kappa)$ that we refer to as the *resonances* of this perturbation problem.

The following was proved in [**4**, §5].

AMS 1969 *subject classifications*. Primary 4748, 8147; Secondary 3577.

Key words and phrases. Perturbation theory, embedded eigenvalues, resonance, Puiseux series, spectral concentration, Schroedinger operator, Auger effect, Fermi's Golden Rule.

[1] This research was supported in part by ARO Grant DA-ARO-D-31-124-G1005.