

ON FINITELY PRESENTED METABELIAN GROUPS

BY GILBERT BAUMSLAG¹

Communicated by George Seligman, September 8, 1971

1. The object of this note is to announce the following:

THEOREM. *Every finitely generated metabelian group can be embedded in a finitely presented metabelian group.*

This theorem should be compared with the example of a 3-generator, 3-relator group with a derived group which is free abelian of infinite rank [2]. It serves to illustrate, in particular, that the normal subgroup structure of (even) finitely presented (metabelian) groups is much more complex than one might have thought.

2. **Sketch of the proof.** Let G be a finitely generated metabelian group.

The first step in the proof is to embed G in a factor group W/N of the wreath product W of two finitely generated abelian groups. This embedding is chosen in such a way that N is contained in the base group of W .

The second, and in fact the main step in the proof, is to embed W in a finitely presented metabelian group \hat{W} in such a way that the normal closure \hat{N} of N in \hat{W} meets W in N :

$$\hat{N} \cap W = N.$$

It follows that G is embedded in \hat{W}/\hat{N} . It follows immediately from a theorem of Philip Hall [1] that every factor group of a finitely presented metabelian group is finitely presented. Thus G has been embedded in the finitely presented metabelian group \hat{G}/\hat{N} .

3. There are continuously many 2-generator center-extended-by-metabelian groups (P. Hall [1]). This means that there are 3-step solvable (indeed, center-extended-by-metabelian) groups, on 2 generators, that cannot be embedded in any finitely presented group whatsoever, let alone in a finitely presented solvable group of derived length 3.

REFERENCES

1. P. Hall, *Finiteness conditions for soluble groups*, Proc. London Math. Soc. (3) **4** (1954), 419–436. MR 17, 344.
2. G. Baumslag, *A finitely presented metabelian group with a free abelian derived group of infinite rank*, Proc. Amer. Math. Soc. (submitted).

DEPARTMENT OF MATHEMATICS, RICE UNIVERSITY, HOUSTON, TEXAS 77001

AMS 1970 subject classifications. Primary 20F05.

Key words and phrases. Finitely presented metabelian group.

¹ Support from the National Science Foundation is gratefully acknowledged.

Copyright © American Mathematical Society 1972