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p-ADIC CURVATURE AND A CONJECTURE OF SERRE

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1. In this note we announce a vanishing theorem for the cohomology of discrete subgroups of p-adic groups. The methods and results bear a striking analogy with the real case (see Matsushima [**4**]). In particular, we define "p-adic curvature" for the p-adic symmetric spaces of Bruhat-Tits (see [**1**]). As in the real case, we then reduce the proof of our vanishing theorem to the assertion that the minimum eigenvalues of certain p-adic curvature transformations are sufficiently large. This last condition can then be verified for "sufficiently large" residue class fields.

Before giving a more detailed description of our results we introduce some notation. Thus let $\boldsymbol{Z}$ denote the ring of rational integers and $\boldsymbol{Q}$, $\boldsymbol{R}$, and $\boldsymbol{C}$ the fields of rational, real and complex numbers, respectively. For a prime p, $\boldsymbol{Q}_p$ will denote the p-adic completion of $\boldsymbol{Q}$. More generally k_v will denote a nondiscrete, totally disconnected, and locally compact (commutative) field.

Let $\boldsymbol{G}$ denote a simply-connected, linear algebraic group defined and simple over k_v and let $\boldsymbol{G}_{k_v}$ denote the k_v-rational points of $\boldsymbol{G}$. Let $V_{\boldsymbol{Q}}$ denote a finite-dimensional vector space over $\boldsymbol{Q}$, Γ an abstract group, and $\rho:\Gamma \to \operatorname{Aut} V_{\boldsymbol{Q}}$ a representation. Let $H^i(\Gamma, \rho)$ denote the ith Eilenberg-Mac Lane group of Γ with respect to ρ. If $V = \boldsymbol{Q}$, and ρ is the trivial representation we write $H^i(\Gamma, \boldsymbol{Q})$ in place of $H^i(\Gamma, \rho)$. By a *uniform lattice* in $\boldsymbol{G}_{k_v}$ we mean a discrete subgroup $\Gamma \subset \boldsymbol{G}_{k_v}$ such that $\boldsymbol{G}_{k_v}/\Gamma$ is compact.

THEOREM 1. *For every integer l, there is an integer $N(l)$ such that if the residue class field of k_v has at least $N(l)$ elements, if $\Gamma \subset \boldsymbol{G}_{k_v}$ is a uniform lattice, and if $\rho:\Gamma \to \operatorname{Aut} V_{\boldsymbol{Q}}$ is a finite-dimensional representation such that $\rho(\Gamma)$ is contained in the orthogonal group of some positive-definite quadratic form on $V_{\boldsymbol{Q}}$, then $H^i(\Gamma, \rho) = 0$ for $0 < i < l$. In particular, $H^i(\Gamma, \boldsymbol{Q}) = 0$ for $0 < i < l$.*

Except for our restriction on the residue class field, this theorem answers a question raised by J.-P. Serre for the trivial representation. It seems likely that sharper estimates for the minimum eigenvalues of p-adic curvature will enable one to eliminate the restriction on the residue class field. For nontrivial ρ, we have had to introduce the additional restriction that $\rho(\Gamma)$ is contained in the orthogonal group of a positive-definite form on $V_{\boldsymbol{Q}}$. Serre has conjectured that if ρ is obtained from a representation of $\boldsymbol{G}_{k_v}$ in

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