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RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited. Manuscripts more than eight typewritten double spaced pages long will not be considered as acceptable. All research announcements are communicated by members of the Council of the American Mathematical Society. An author should send his paper directly to a Council member for consideration as a research announcement. A list of the members of the Council for 1972 is given at the end of this issue.

NORMAL SOLVABILITY AND ϕ-ACCRETIVE MAPPINGS OF BANACH SPACES[1]

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Let X and Y be real Banach spaces. A mapping f of X into Y is said to be *normally solvable* if $f(X)$ is closed in Y. The theory of normal solvability uses this property together with infinitesimal assumptions upon the structure of $f(X)$ to obtain conclusions upon the global structure of $f(X)$, and in particular the conclusion that $f(X)$ is all of Y or that a given element y_0 of Y lies in $f(X)$.

It is our purpose in the present note to present some new and sharper results in this theory, and to apply these results to the proof of existence theorems for equations of the form $f(x) = y$ for mappings f which lie in a general class of ϕ-accretive mappings, generalizing the concept of a monotone mapping from X to X^* and of an accretive mapping from X to X.

DEFINITION 1. *Let X and Y be real Banach spaces, Y^* the conjugate space of Y. Let ϕ be a mapping of X into Y^* such that $\phi(X)$ is dense in Y^* with*

$$\|\phi(x)\|_{Y^*} = \|x\|_X, \qquad \phi(\xi x) = \xi\phi(x),$$

for all x in X, $\xi \geqq 0$. Then:

(a) *A mapping f of X into Y is said to be ϕ-accretive if, for all x and u in X,*

$$(f(x) - f(u), \phi(x - u)) \geqq 0.$$

(b) *The map f is said to be strongly ϕ-accretive if there exists $c > 0$ such that, for all x and u in X,*

$$(f(x) - f(u), \phi(x - u)) \geqq c\|x - u\|^2.$$

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