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BOOK REVIEWS

Abstract Harmonic Analysis; I, II, by E. Hewitt and K. A. Ross. Springer-Verlag, New York, Heidelberg, Berlin (1963; 1969).

Abstract harmonic analysis is concerned with the theory of Fourier series and integrals in the context of topological groups.

As would be expected even from such a summary description "abstract" theories and "classical" theories are intimately connected. Abstract harmonic analysis cannot replace classical Fourier analysis but it is now almost impossible to work in the latter subject without having in mind the "abstract" developments.

The relationship between classical theories and new theories however is by no means clear. For one thing there is no general agreement as to the degree of generality in which it is most fruitful to work for each problem. Following the discovery of an invariant measure on all locally compact topological groups, and with the development of the theory of Banach algebras, it became apparent that a large portion of classical harmonic analysis could be done in the context of locally compact abelian (LCA) groups.

The very satisfactory duality theory for LCA groups and the existence of powerful structure theorems contributed to make this development possible. The theorems of Katznelson (and others) on functions that operate in $L^1(G)$, the proof, by Malliavin, of the impossibility of spectral synthesis, the characterization established by P. J. Cohen of the idempotent measures were all conceived in the context of LCA groups. Even the classical theory of functions on the disk was fruitfully extended to the case of LCA groups, with appropriate additional hypotheses.

These developments are described in W. Rudin's monograph, *Fourier analysis on groups*, which appeared in the early sixties and provided further stimulus for research in this area.

The situation is different for harmonic analysis on noncommutative groups. Duality theory for nonabelian groups is based on the concept of irreducible representations, but on the other hand, the task of describing the irreducible representations of noncommutative groups, even finite groups, seems formidable.

Nevertheless, a considerable amount of research has been done in Fourier analysis on noncommutative groups. Some researchers are concerned with particular groups, such as $SL(2)$ and $U(2)$, of which the representations are carefully analyzed and described in order to develop the theory; but others are attempting to develop general methods which will

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