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SMOOTH S^1 ACTIONS ON HOMOTOPY COMPLEX PROJECTIVE SPACES AND RELATED TOPICS[1]

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This paper is dedicated to Professors Leroy M. Kelly and Fritz Herzog who gave so enthusiastically of their time and talent in developing undergraduate mathematicians at Michigan State University. I was one of their beneficiaries.

0. **Introduction and motivation.** We begin by listing some questions and remarks which establish the theme of this paper.

1. Which cobordism classes of oriented manifolds admit nontrivial circle actions? Answer: Atiyah-Hirzebruch [**4**]: For a compact oriented manifold X of dim $4k$, its $\hat{\mathscr{A}}$ genus vanishes iff there is a multiple mX which is cobordant to Y, with $W_2(Y) = 0$, which admits a nontrivial circle action on each of its components. The $\hat{\mathscr{A}}$ genus is the genus belonging to the power series $(x/2)(\sinh x/2)^{-1}$.

2. Which manifolds in a given homotopy type admit nontrivial circle actions? More specifically, of those manifolds homotopy equivalent to complex projective n space, which admit nontrivial S^1 actions?

Strong conjecture. If $h: X \to CP^n$ is an orientation preserving homotopy equivalence and if X supports a nontrivial circle action then $h^*\hat{\mathscr{A}}(CP^n) = \hat{\mathscr{A}}(X)$ where

$$\hat{\mathscr{A}}(X) = \prod (x_i/2)(\sinh x_i/2)^{-1} \in H^*(X, Q)$$

and the elementary symmetric functions of the x_i^2 give the Pontrjagin classes of X. In other words, the homotopy equivalence must preserve the total $\hat{\mathscr{A}}$ cohomology class.

Weak conjecture. To the hypothesis of the strong conjecture add the condition that the fixed point set of the action consists of isolated fixed points. Then

$$h^*\hat{\mathscr{A}}(CP^n) = \hat{\mathscr{A}}(X).$$

A corollary of the strong conjecture is that most homotopy complex projective spaces do not admit S^1 actions. The weak conjecture is discussed in detail in Part II, §2.

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