BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 1, January 1972

ON THE CLASS NUMBERS OF TOTALLY IMAGINARY QUADRATIC EXTENSIONS OF TOTALLY REAL FIELDS

BY JUDITH E. SUNLEY[1]

Communicated by Oscar Goldman, June 22, 1971

ABSTRACT. Let K be an arbitrary totally real algebraic number field of degree $n \geqq 2$. It is shown that there exists an upper bound on the absolute value of the discriminant of any totally imaginary quadratic extension of K of class number h with at most one possible exception. This bound depends in an effective way on the parameters of the field K.

1. **Introduction.** The problem of determining all imaginary quadratic fields of class number h has long been of importance in number theory. In recent years this problem has been solved for $h = 1$ by Stark [**8**] and for $h = 2$ by Goldstein [**3**], Baker [**1**], and Stark [**9**]. So far no real progress has been made for any other value of h.

This note is an announcement of research which extends some of the known results on class numbers of imaginary quadratic fields to the case of totally imaginary quadratic extensions of a totally real field. The main result is the following.

THEOREM 1. *Let K be an arbitrary totally real algebraic number field, h an arbitrary positive integer. With at most one possible exception, all totally imaginary quadratic extensions L of K with class number h satisfy*

$$|d_L| < C(K, h)$$

where $C(K, h)$ is an effectively computable constant and d_L is the discriminant of L.

This result is a generalization of similar theorems due to Heilbronn and Linfoot [**4**] for $h = 1$ and Tatuzawa [**11**] for arbitrary h, both results in the special case where $K = \boldsymbol{Q}$ and L is an imaginary quadratic field. A sketch of the proof will be given here. The details will appear elsewhere.

2. **The estimate for** $\Pi(x, \chi)$. Let $\Pi(x) = \sum_{N\mathfrak{A} \leqq x} 1$, and let $\Pi(x, \chi) = \sum_{N\mathfrak{A} \leqq x} \chi(\mathfrak{A})$ where $\mathfrak{A}$ runs over all integral ideals of some algebraic number field K. In the case $K = \boldsymbol{Q}$ there is a classical result of Pólya [**6**] which says that $|\Pi(x, \chi)| < l^{1/2} \log l$, where l is the period of the character χ. At the same time Pólya's result appeared, Landau [**5**] obtained an extension of the result to the case where the degree of K is at least two and χ is an ideal

AMS 1969 *subject classifications*. Primary 1065; Secondary 1068.
Key words and phrases. Number field, class number, imaginary quadratic extension.

[1] This research was partially supported by NSF Science Development Grant GU-2061 and comprised the author's doctoral dissertation at the University of Maryland.