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RIGIDITY THEOREMS FOR SURFACES IN EUCLIDEAN SPACE

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Let M be a surface immersed in euclidean m-space E^m, and let ∇ and ∇' be the covariant differentiations of M and E^m respectively. Let u and v be two tangent vector fields on M. Then the second fundamental form h is given by

$$\nabla'_u v = \nabla_u v + h(u, v). \tag{1}$$

If $e_1, e_2, e_3, \ldots, e_m$ is a local field of orthonormal frame such that e_1, e_2 are tangent to M and $e_3, \ldots, e_m$ are normal to M, then the mean curvature vector H is given by

$$H = \frac{1}{2}\sum_{i=1}^{2} h(e_i, e_i). \tag{2}$$

For a normal vector field η and a tangent vector field u on M, let $\nabla^*_u\eta$ denote the normal component of $\nabla'_u\eta$. Then ∇^* defines a connection in the normal bundle of M in E^m. A normal vector field η is said to be parallel in the normal bundle if $\nabla^*\eta = 0$. Let h^r_{ij}, $i, j = 1, 2, r = 3, \ldots, m$, be the coefficients of the second fundamental form h. Then the Gauss curvature K and the normal curvature K_N are given by

$$K = \sum_{r=3}^{m} (h^r_{11}h^r_{22} - h^r_{12}h^r_{12}), \tag{3}$$

$$K_N = \sum_{r,s=3}^{m} \left[\sum_{k=1}^{2} (h^r_{1k}h^s_{2k} - h^r_{2k}h^s_{1k}) \right]^2, \tag{4}$$

respectively. The mean curvature vector H, the Gauss curvature K, and the normal curvature K_N play important roles, in differential geometry, for surfaces in euclidean space.

Let $\langle\ ,\ \rangle$ denote the scalar product of E^m. If the mean curvature vector H is nowhere zero and there exists a function f on M such that $\langle h(u, v), H\rangle = f\langle u, v\rangle$ for all tangent vector fields u, v on M, then M is called a *pseudo-umbilical surface* of E^m.

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