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EXTENDING FOURIER TRANSFORMS INTO SZ.-NAGY-FOIAŞ SPACES[1]

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1. **Introduction.** Let $s(z)$ be a function in the unit ball of H^∞ of the unit disc. Let $\Delta(e^{it}) = (1 - |s(e^{it})|^2)^{1/2}$ and let $E = \{t|\Delta(e^{it}) > 0\}$. We consider the subspace $\mathcal{M}_s$ of $H^2 \oplus L^2(E)$ of pairs of the form $(s(z)f(z), \Delta(e^{it})f(e^{it}))$ for $f \in H^2$. The *Sz.-Nagy-Foiaş space associated with s* is the orthogonal complement $\mathcal{M}_s^\perp = [H^2 \oplus L^2(E)] \ominus \mathcal{M}_s$. The function s is *inner* precisely when E is a zero set, and in that case $\mathcal{M}_s$ reduces to the invariant subspace sH^2.

In the latter case, two "Fourier transforms" have recently been defined, from various L^2 spaces into $\mathcal{M}_s^\perp = H^2 \ominus sH^2$. The first such unitary operator, defined by Ahern and Clark [**1**] and Kriete [**3**] is obtained as follows. Let σ be a singular measure without atoms on $[0, 2\pi]$ and set

$$(1)\qquad s_\lambda(z) = \exp\left[-\int_0^\lambda (e^{i\theta} + z)/(e^{i\theta} - z)\, d\sigma(\theta)\right], \qquad s(z) \equiv s_{2\pi}(z).$$

An operator $\mathcal{U}$ from $L^2(d\sigma)$ to $\mathcal{M}_s^\perp$ is defined by

$$(\mathrm{A})\qquad (\mathcal{U}f)(z) = 2^{1/2}\int_0^{2\pi} f(\lambda)s_\lambda(z)(1 - e^{-i\lambda}z)^{-1}\, d\sigma(\lambda).$$

Then $\mathcal{U}$ is unitary and satisfies

$$(2)\qquad \mathcal{U}^*T\mathcal{U} = (I - K)M,$$

where T is the restricted shift on $\mathcal{M}_s^\perp$:

$$Tg = P_{\mathcal{M}_s^\perp} zg$$

and, for $f \in L^2(d\sigma)$,

$$(3)\qquad Mf = e^{it}f(t), \qquad Kf = 2\int_0^t e^{-\sigma([\lambda,t])}f(\lambda)\, d\sigma(\lambda).$$

The second transform was defined by the author in [**2**]. Let ν be an arbitrary singular measure on $[0, 2\pi]$ and let

$$(4)\qquad s(z) = \left[\int_0^{2\pi} \frac{e^{i\theta} + z}{e^{i\theta} - z}\, d\nu(\theta) - 1\right]\left[\int_0^{2\pi} \frac{e^{i\theta} + z}{e^{i\theta} - z}\, d\nu(\theta) + 1\right]^{-1}.$$

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