BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 1, January 1972

BOOK REVIEWS

Perceptrons, An Introduction to Computational Geometry, by Marvin Minsky and Seymour Papert. MIT Press, Cambridge, Mass., 1969.

This book is a very interesting and penetrating study of the power of expression of perceptrons and some other mathematical problems concerning memory and learning. This subject is still quite new and hence at a stage of development in which the most important discoveries are being done. It seems to differ from the theory of automata in its greater relevance to our ideas about the organization of the brain and the construction of "such" machines.

The main positive result which motivates the research presented in this book is *the perceptron learning theorem* (proved in Chapter 11). This theorem was discovered as a property of Rosenblatt's perceptrons (the history is given in the book) but in fact it belongs to general linear approximation theory and can be stated as follows.

Let H be a real Hilbert space, $F^+, F^- \subseteq H, F^+ \cap F^- = \emptyset$ and for every $x \in F^+ \cup F^-$ we have $\delta \leqq \|x\| \leqq M$, where $\delta > 0$. We assume moreover that F^+ and F^- are *linearly separable with resolution* δ, i.e. there exists an $a^* \in H$ with $\|a^*\| = 1$ such that

$$(a^*, x) \geqq \delta \quad \text{if } x \in F^+ \quad \text{and} \quad (a^*, x) \leqq -\delta \quad \text{if } x \in F^-.$$

Given any sequence $x_1, x_2, \ldots \in F^+ \cup F^-$ and any $\varepsilon \geqq 0$ we define by induction a sequence of vectors $a_0, a_1, \ldots \in H$ and a sequence of integers $k_1, k_2, \ldots$. We put $a_0 = 0$ and

$$a_{i+1} = a_i + k_{i+1}x_{i+1}, \tag{1}$$

where k_{i+1} is the unique integer with minimal absolute value such that if $x_{i+1} \in F^+$ then

$$(a_i, x_{i+1}) + k_{i+1}\|x_{i+1}\|^2 > \varepsilon.$$

and if $x_{i+1} \in F^-$ then

$$(a_i, x_{i+1}) + k_{i+1}\|x_{i+1}\|^2 < -\varepsilon.$$

Then the sequence $k_1, k_2, \ldots$ *has finitely many terms different from* 0 *and moreover*

$$\sum_{i=1}^{\infty} |k_i| \leqq (M^2 + 2\varepsilon)/\delta^2.$$

(In the book $\varepsilon = 0$ and $M = 1$, but the proof is almost the same for all $\varepsilon \geqq 0$ and $M \geqq \delta$.)