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BSJ DOES NOT MAP CORRECTLY INTO *BSF* MOD 2

BY ROBERT CLOUGH AND JAMES STASHEFF

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It has been widely conjectured that there is a homotopy commutative diagram

$$\text{(A)}\qquad \begin{array}{ccc} BSO & \xrightarrow{\;J\;} & BSF \\ & {\scriptstyle j}\searrow \quad \nearrow & \\ & BSJ & \end{array}$$

where J is the stable Whitehead J-homomorphism and BSJ is the space constructed in [1]. Indeed, in [5], Quillen proves the Adams conjecture, which implies that SJ maps into SF mod 2 in a way consistent with diagram (A). In [6], Sullivan proves that SF even splits mod 2 into $SJ \times \mathrm{Coker}(J)$, although this splitting does not necessarily deloop to a splitting of BSF. In [3], Madsen proves that diagram (A), if it exists, does not deloop twice.

Our purpose is to sketch a proof that diagram (A) does not exist. Details will follow in [2].

THEOREM. *It is impossible to define Stiefel-Whitney classes w_n for $n \geqq 2$ in $H^n(BSJ; Z_2)$ in such a way that all of the following conditions hold*:

(1) $j^* w_n = w_n \in H^n(BSO; Z_2)$;

(2) *the w_n satisfy the Wu formulas.*

COROLLARY. *Diagram* (A) *does not exist* mod 2.

SKETCH OF PROOF OF THEOREM. We assume for a contradiction that Stiefel-Whitney classes can be chosen satisfying conditions (1) and (2).

By [1], we have

$$H^*(BSJ; Z_2) = P[w_2, w_3, \cdots] \otimes E[e_3, e_4, \cdots]$$

and BSJ is the base of the 2-primary fibration

$$BSO \xrightarrow{\psi^3 - 1} BSO \xrightarrow{j} BSJ.$$

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