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EXISTENCE OF POLYNOMIAL IDENTITIES IN $A \otimes_F B$

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ABSTRACT. The following theorem is proved: If A, B are PI-algebras over a field F, then $A \otimes_F B$ is also a PI-algebra.

Let F be a field, A and B two PI-algebras (i.e., algebras satisfying a polynomial identity) over F. The problem whether also $A \otimes_F B$ satisfies a polynomial identity has been open for some time [**1**, p. 228]. We have proved that if A and B are PI-algebras, then $A \otimes_F B$ is indeed a PI-algebra. A very brief outline of the proof is given here, and the details of the proof will appear elsewhere.

Let $\{x\}$ be an infinite set of noncommutative indeterminates over F, and let $F[x]$ be the free ring in $\{x\}$ over F. Let $\{x_1, x_2, \cdots\} = \{x_\nu\} \subseteq \{x\}$ be a fixed countable sequence of indeterminates from $\{x\}$. Let S_n denote the group of all permutations of $\{1, \cdots, n\}$ and let

$$V_n = \operatorname{span}\{x_{\sigma_1} \cdots x_{\sigma_n} \mid \sigma \in S_n\}$$

be the $n!$ dimensional vector space, spanned by the $n!$ monomials $x_{\sigma_1} \cdots x_{\sigma_n}$ $(\sigma \in S_n)$ in $x_1, \cdots, x_n$.

An ideal $Q \subseteq F[x]$ is a T-ideal if $f(x_1, \cdots, x_n) \in Q$ and $g_1, \cdots, g_n \in F[x]$ implies that $f(g_1, \cdots, g_n) \in Q$. It is well known [**1**, p. 234] that the set of all identities of a PI-algebra is a T-ideal. Let Q be the T-ideal of identities of a PI-algebra A. For each integer $0 < n$, define $d_n = \dim(V_n/(Q \wedge V_n))$. We call $\{d_\nu\}$ "the sequence of codimensions" of Q (or A). Codimensions play an important role in the proof that $A \otimes_F B$ is a PI-algebra.

It follows from the definition of d_n that there exist d_n monomials $M_1(x_1, \cdots, x_n), \cdots, M_{d_n}(x_1, \cdots, x_n)$ which span V_n modulo Q, i.e., for each $\sigma \in S_n$ there exist coefficients $\phi_i(\sigma) \in F$, $1 \leqq i \leqq d_n$, such that

$$M_\sigma(x) = x_{\sigma_1} \cdots x_{\sigma_n} \equiv \sum_{i=1}^{d_n} \phi_i(\sigma) M_i(x) \pmod{Q}.$$

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