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REPRESENTATION THEORY FOR DIVISION ALGEBRAS OVER LOCAL FIELDS (TAMELY RAMIFIED CASE)

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Aside from intrinsic interest there are three (related) reasons for studying the unitary representations of p-adic division algebras.

(1) They provide heuristics for more difficult (e.g., noncompact) p-adic groups.

(2) They would be basic building blocks in a theory of representations of reductive algebraic p-adic groups based on the philosophy of cusp forms.

(3) There seem to be deep relations of representations of division algebras with representations of Gl_n, having implications in the theory of automorphic forms. This was pointed up in Jacquet-Langlands [2].

We announce here, for the tamely ramified case, a classification of the representations (Theorem 1), and a result related to (3) (Theorem 2). I would like to thank R. P. Langlands for some stimulating conversations, and in particular for telling me of the likelihood of Theorem 2.

Let F be a locally compact non-archimedean field of residual characteristic p. Let R be its maximal order, π a prime element, $F^\times$ its multiplicative group, and $U=1+\pi R\subseteq F^\times$. Let D be a division algebra over F. Let S, Π, $D^\times$, V be its maximal order, and so forth. We will say D is tamely ramified if its degree, n, is prime to p. This is the same as to say all its commutative subfields are tamely ramified over F.

Let F' be a finite extension, with maximal order R', prime π', $U'=1+\pi'R'$ and multiplicative group $F'^\times$. Let $N(F'/F): F'^\times\to F^\times$ be the norm map. Let ψ be a character of $F'^\times$ and $A\subseteq F'^\times$ a subgroup. We will say ψ is nondegenerate on A if there is no proper subextension F'', $F\subseteq F''\subset F'$, such that $\ker N(F'/F'')\cap A\subseteq \ker \psi\cap A$. Suppose now F' is tamely ramified over F. We will say a character ψ of $F'^\times$ is admissible if

(a) ψ is nondegenerate on $F'^\times$, and

(b) if on U', $\psi=\psi''\circ N(F'/F'')$, where ψ'' is nondegenerate on $U''\subseteq F''^\times$, then F' is unramified over F''.

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