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# ON CONJUGATE POWERS IN EIGHTH-GROUPS

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Let $G$ be a finitely presented group with generating elements $a_1, \cdots, a_\lambda$ and defining relations $R_1=1, \cdots, R_\mu=1$. We assume without loss of generality that the *relators* $R_i$ form a symmetric set, i.e. that the $R_i$ are cyclically reduced and are closed under the operations of taking inverses and cyclic transforms. We call $G$ an *eighth-group* if it satisfies the following condition:

(*) If $R_i \cong XY$ and $R_j \cong XZ$ are distinct relators, then the length of the common initial segment $X$ is less than 1/8 the length of either relator.

A classical example of such a group is the fundamental group $G_k$ of an orientable closed 2-manifold of genus $k>2$; it has the presentation

$$G_k = \text{gp}(a_1, b_1, \cdots, a_k, b_k; a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_k b_k a_k^{-1} b_k^{-1} = 1).$$

More generally, the Fuchsian groups $F(p; n_1, \cdots, n_d; m)$, see Greenberg [3], are eighth-groups if $4p+d+m, n_1, \cdots, n_d > 8$.

The class of eighth-groups were first considered by Greendlinger who solved the word problem [4] and the conjugacy problem [5] for them. Similar "small cancellation" groups have been studied by Tartakovskii [8], Britton [1], Lyndon [6] and Schupp [7], among others.

We now state our main result.

THEOREM. *Suppose $W$ is an element of infinite order in an eighth-group $G$. If $|m| \neq |n|$ then $W^m$ and $W^n$ are in different conjugacy classes. In particular, $W$, $W^2$, $W^3$, $\cdots$ are in different conjugacy classes.*

This theorem has already been known to hold for the above fundamental groups $G_k$ and for the Fuchsian groups; but all the proofs have been topological. Our theorem holds for a much wider class of groups and, moreover, the proof is purely algebraic.

The author conjectures that the theorem also holds for the small concellation groups in general.

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