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CURVATURE AND COMPLEX ANALYSIS

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In this note, we announce some results in the geometric theory of several complex variables. For the first theorem, recall the theorem of Cartan-Hadamard: if M is a Riemannian manifold with nonpositive Riemannian curvature, complete and simply connected, then it is diffeomorphic to Euclidean space. When the metric is actually Kähler, the following result gives additional information:

THEOREM 1. *Let M be a complete simply connected Kähler manifold with nonpositive Riemannian curvature. Then*

(i) *M is a Stein manifold.*

(ii) *If ρ denotes the distance function from a fixed point $0 \in M$, then* $\log \rho$ *is plurisubharmonic and* ρ^2 *and* $\log(1+\rho^2)$ *are both* C^∞ *and strictly plurisubharmonic. In fact*

$$dd^c\rho^2 \geqq 4\omega, \qquad dd^c \log(1+\rho^2) \geqq 4\omega/(1+\rho^2)^2$$

where $d^c = (-1)^{1/2}\,(d''-d')$ *and* ω *is the Kähler form of M.*

(iii) *If Riemannian curvature* $\leqq -c^2 < 0$, *then* $dd^c\rho^2 \geqq (2+2c\rho \coth c\rho)\omega$, $dd^c \log\,(1+\rho^2) \geqq \alpha\omega$, *where* coth *denotes the hyperbolic cotangent and* $\alpha = \min\,\{2, c \coth c - 1\} > 0$.

(iv) *If* $-d^2 \leqq$ *Riemannian curvature* $\leqq 0$, *then*

$$dd^c\rho^2 \leqq (4\rho d \coth \rho d + 2)\omega,$$

$$dd^c \log\,(1+\rho^2) \leqq (1/(1+\rho^2))(4\rho d \coth \rho d + 2)\omega.$$

Part (i) is a known result. See [4].

For the next theorem, we recall that it is generally conceded that no holomorphic function on $\mathbf{C}^n$ can be in L_p, $p \leqq \infty$. In transferring this theorem to Kähler manifolds, it is obviously necessary to forego the case of $p = \infty$.

THEOREM 2. *Let M be an n-dimensional complete simply connected Kähler manifold with nonpositive Riemannian curvature. Then*

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