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KREISS' MIXED PROBLEMS WITH NONZERO INITIAL DATA

BY JEFFREY RAUCH[1]

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In [3], Kreiss has shown that a large class of mixed initial boundary-value problems of hyperbolic type are well-posed in the $\mathcal{L}_2$ sense. However only zero initial data were considered. For the same class of problems we show that if square integrable initial data are prescribed then there is a unique solution which is square integrable for each positive time.

The differential operators under consideration are of the form

$$Lu = \frac{\partial u}{\partial t} - \sum_{j=1}^{n} A_j(t, x) \frac{\partial_j u}{\partial x_j} + B(t, x)u$$

where u is a complex k-vector, and A_j and B are $k \times k$ matrix-valued functions. The operator (L) is assumed strictly hyperbolic, that is $\sum A_j \xi_j$ has k distinct real eigenvalues for each $\xi \in R^n \backslash 0$. The coefficients are assumed to be smooth functions which are constant outside a compact set. In addition, we require that det $A_1 \neq 0$ when $x_1 = 0$. The following notation is employed:

$$\begin{aligned} \Omega &= \{x \in R^n \mid x_1 \geqq 0\}, \\ \partial\Omega &= \text{boundary of } \Omega = \{x \in R^n \mid x_1 = 0\}, \\ x &= (x_1, x') = (x_1, x_2, \cdots, x_n). \end{aligned}$$

Boundary conditions are prescribed with the aid of a boundary operator $M(t, x')$ which is a smooth $l \times k$ matrix-valued function where l = number of negative eigenvalues of A_1. We suppose that M has rank l and is independent of t, x' for $|t| + |x'|$ large.

The basic problem is to show that for given

$$F \in \mathcal{L}_2([0, T] \times \Omega), \qquad g \in \mathcal{L}_2([0, T] \times \partial\Omega), \qquad f \in \mathcal{L}_2(\Omega).$$

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