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AN EXACT SEQUENCE INVOLVING THE CHERN CHARACTER

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In [3] the author defined maps $b_n' : U(n) \to \Omega^2 U(n+1)$ which are deformations of the classical Bott homotopy equivalence $b: U \to \Omega^2 U$ [1], i.e., the composite $U(n) \to \Omega^2 U(n+1) \to \Omega^2 U$ is homotopic to the composite $U(n) \to U \to \Omega^2 U$. The maps b_n' are natural with respect to the inclusions $U(k) \subset U(n)$ for $k \leqq n$. The maps b_n' may be used to define homomorphisms $B_n: \pi_r(U(n)) \to \pi_{r+2}(U(n+1))$ as the composite homomorphism

$$\pi_r(U(n)) \xrightarrow{b'_{n*}} \pi_r(\Omega^2 U(n+1)) \xrightarrow{\partial^{-2}} \pi_{r+2}(U(n+1)).$$

The advantage gained by using the maps B_n is that they give information on the nonstable homotopy of $U(n)$ not available from the classical Bott maps, and they agree with the classical results in the stable range. For example, the results of [3] show that the map $B_n: \pi_r(U(n)) \to \pi_{r+2}(U(n+1))$ is an isomorphism for $r \leqq 2n-1$, and $B_n: \pi_{2n}(U(n)) \to \pi_{2(n+1)}(U(n+1))$ is a monomorphism. Kenneth Millett has calculated $B_n: \pi_{2(n+r)}(U(n)) \to \pi_{2(n+r+1)}(U(n+1))$ for $r=2, 3$.

The purpose of this announcement is to describe an application of the maps b_n' to complex K-theory. We work throughout in the category of finite CW complexes with basepoint. We use Q to denote the additive group of rational numbers, and Z to denote the group of integers.

1. **The spectrum TU.** We use the maps b_n' to define a spectrum TU by setting $TU_{2k} = \Omega U(k)$, $TU_{2k+1} = U(k)$ for $k \geqq 0$, and $TU_m = \text{point}$ for $m < 0$. The maps of the spectrum are

$$\tau_{2k} = \mathrm{id}: TU_{2k} = \Omega U(k) \to \Omega U(k) = \Omega TU_{2k+1}$$

and

$$\tau_{2k+1} = b_k' : TU_{2k+1} = U(k) \to \Omega^2 U(k+1) = \Omega TU_{2k+2}.$$

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