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AN EXTENSION OF KHINTCHINE'S ESTIMATE FOR LARGE DEVIATIONS TO A CLASS OF MARKOV CHAINS CONVERGING TO A SINGULAR DIFFUSION[1]

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1. For $S(n)$ a sum of n independent identically distributed random variables with mean zero and variance one, Khintchine obtained the estimate

$$P(S(n)/\sqrt{n} \geqq \alpha_n) = \exp\left(-\tfrac{1}{2}\alpha_n^2(1+o(1))\right) \tag{1.1}$$

where $\alpha_n \uparrow +\infty$ with a certain rate of growth. Recently an elementary proof of this estimate was given by Mark Pinsky in [4]. Using Pinsky's method and some nontrivial estimates in the theory of partial differential equations we prove that (1.1) holds for an interesting class of Markov chains converging to a singular diffusion process on the half line $\bar{R}_+ = [0, \infty]$. The random walks we shall study have state space $I^+ = \{0, 1, 2, \cdots\}$ and transition probabilities $p(i, j)$ given by $p(i, i) = 0$, $i = 0, 1, 2, \cdots$,

$$\begin{aligned} p(i, i+1) &= \tfrac{1}{2}(1+\gamma/i), \\ p(i, i-1) &= 1 - p(i, i+1), \\ p(0, 1) &= 1, \quad \text{the “reflecting barrier condition” at the origin.} \end{aligned} \tag{1.2}$$

In addition we assume $0 \leqq \gamma < 1$.

$\{X(n): n = 0, 1, \cdots,\}$ denotes the random walk with transition matrix $p(i,j)$ defined by (1.2), and $P_x(\ )$ denotes the measure induced on sequences of nonnegative integers by $\{X(n): X(0) = x\}$.

THEOREM 1.1. *If $\{\alpha_n\}$ is any sequence increasing to $+\infty$ satisfying the condition $\lim_{n\to\infty} \alpha_n^2 - (\log n)/2 = -\infty$, then for each $\epsilon > 0$ there exists an integer $N(\epsilon)$ so that for $n \geqq N(\epsilon)$ we have*

$$\exp\left(-\tfrac{1}{2}\alpha_n^2(1+\epsilon)\right) \leqq P_0(X(n)/\sqrt{n} \geqq \alpha_n) \leqq \exp\left(-\tfrac{1}{2}\alpha_n^2(1-\epsilon)\right). \tag{1.3}$$

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