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CONVENIENT CATEGORIES OF TOPOLOGICAL ALGEBRAS

BY EDUARDO J. DUBUC AND HORACIO PORTA[1]

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Introduction. Concrete associative algebras with a topology have long arisen in mathematical practice; thus, a notion of topological space with algebraic operations making it an associative algebra was in order. The subject naturally evolved into the present general theory of abstract topological algebras [5]. Classes of such objects (together with continuous maps respecting the algebraic structure) form categories which, understandably, do not share some important properties of their purely algebraic analogues. Specifically, *their relation with the base category* S *of sets is not satisfactory.* This is essentially due to the fact that taking forgetful functors into S is forgetting too much. Also, the set of morphisms between any two such algebras naturally carries a topology which is inherited from the topologies of the algebras, and which is not taken into account (it is ignored) by the representable functors landing in S.

The category of topological spaces is actually the natural *base* category (that is, the place where the forgetful and representable functors land) for a categorical approach to the study of classes of topological algebras. However, this category is not "set-like" enough to make such an approach possible.

Categories which, like S, have enough structure to serve as base categories have been recognized by category theorists during the sixties ([1], [4]) when the concept of *closed* category was developed. Compactly generated topological spaces form such a convenient (closed) category [8].

We introduce here a systematic treatment of categories of topological algebras considered as categories based on the category K *of compactly generated Hausdorff spaces.*

This leads to the definition of *K-topological algebras.* Roughly, a K-topological algebra is a complex algebra with a topology making the operations continuous when restricted to compact subsets. This is a broad class of algebras, containing *all* algebras with jointly continuous product, but failing to contain some topological algebras with

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