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PFISTER FORMS AND K-THEORY OF FIELDS[1]

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Let F be a field of characteristic different from two. We shall write $W(F)$ to denote the Witt ring of F, and $I(F)$ to denote the ideal of all even dimensional forms in $W(F)$. Also, let K_nF ($n \geqq 1$) be the higher K-groups of F defined by Milnor in [3], and $k_nF = K_nF/2K_nF$. The elements $l(a_1) \cdots l(a_n) \in k_nF$ will be called the *generators* of k_nF. If $(a_1, \cdots, a_n)$ is an n-tuple of nonzero elements of F, we shall write $\langle\langle a_1, \cdots, a_n\rangle\rangle$ for the 2^n-dimensional quadratic form $\otimes_{i=1}^n \langle a_i, 1\rangle$, and refer to it as an *n-fold Pfister form*. Clearly, these n-fold Pfister forms give a system of generators of I^nF as an ideal in $W(F)$. In [3], Milnor showed that

$$s_n(l(a_1) \cdots l(a_n)) = \langle\langle -a_1, \cdots, -a_n\rangle\rangle \pmod{I^{n+1}F}$$

gives a well-defined epimorphism from k_nF onto $I^nF/I^{n+1}F$. In §4 of [3], Milnor raised the question whether these maps are isomorphisms.

In studying this problem, the Pfister forms clearly play a crucial role. In this note, we announce the following results.

THEOREM 1. *The following statements are equivalent*:

(1) $\langle\langle -a_1, -a_2\rangle\rangle$, $\langle\langle -b_1, -b_2\rangle\rangle$ *are isometric* (*Pfister*) *forms.*

(2) $\langle\langle -a_1, -a_2\rangle\rangle \equiv \langle\langle -b_1, -b_2\rangle\rangle \pmod{I^3F}$.

(3) $\left(\dfrac{a_1, a_2}{F}\right)$, $\left(\dfrac{b_1, b_2}{F}\right)$ *are isomorphic* (*quaternion*) *algebras.*

(4) $l(a_1)l(a_2) = l(b_1)l(b_2)$ *in* k_2F.

This theorem shows that $l(a_1)l(a_2) \in k_2F$ or the quaternion algebra

$$\left(\frac{a_1, a_2}{F}\right)$$

(in the Brauer group of F) gives a complete invariant for the isometry class of a 2-fold Pfister form $\langle\langle -a_1, -a_2\rangle\rangle$. One therefore naturally asks if an analogous result will hold for the isometry class of an

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