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BESSEL POTENTIALS. INCLUSION RELATIONS AMONG CLASSES OF EXCEPTIONAL SETS

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1. Let $g_\alpha = g_\alpha(x)$ be the Bessel kernel of order α, $0<\alpha<+\infty$, on R^n; g_α is the Fourier transform of $(2\pi)^{-n/2}(1+|\zeta|^2)^{-\alpha/2}$. For $1<p<\infty$, we define a capacity $B_{\alpha,p}$ (referred to as Bessel capacity): for $A \subset R^n$,

$$B(A) = B_{\alpha,p}(A) = \inf_f \int f(x)^p \, dx$$

the infimum being taken over all functions f in $L_p^+ = L_p^+(R^n)$—positive functions in the Lebesgue class—such that $g_\alpha * f(x) \geqq 1$ for all $x \in A$. The capacities $B_{\alpha,p}$ have been studied extensively in [4]. It is an easy consequence of the definition of $B_{\alpha,p}$ that: $B_{\alpha,p}(A)=0$ if and only if there is an $f \in L_p^+$ such that $g_\alpha * f(x) = +\infty$ on A.

Variants of the Bessel capacities occur for instance in [1], [3], [5].

Our purpose here is to announce results on the relations between the B's for various pairs (α, p). We say that the Bessel capacity B is *stronger* than the Bessel capacity B' (written $B' \preceq B$) if $B(A)=0$ always implies $B'(A)=0$. If in addition, there is a set A such that $B(A)>0$ but $B'(A)=0$ we say B is *strictly stronger* than B' ($B' \prec B$). These are the *relations* between B and B'. If both $B' \preceq B$ and $B \preceq B'$ hold, we say B is *equivalent* to B' ($B \sim B'$). In addition to the relations among the B's, we also give some results concerning relations between Bessel capacities, Hausdorff measures, and classical capacities (C_k below). These classical capacities can be viewed as a special case of general L_p-capacities of [4] when $p=1$ or $p=2$.

Notation. wei B = weight of $B_{\alpha,p} = \alpha p$; ord B = order of $B_{\alpha,p} = \alpha$; ind B = index of $B_{\alpha,p} = (\alpha, p)$. By $(\alpha, p) \prec (\beta, q)$ we shall mean either $\alpha p < \beta q$, or $\alpha p = \beta q$ and $\alpha < \beta$.

2. The principal result is

THEOREM 1. *If B and B' are two Bessel capacities,*
(i) $B' \prec B$ *if and only if* ind $B' \prec$ ind B *and* wei $B' \leqq n$.
(ii) $B' \sim B$ *if and only if* ind B' = ind B *or* wei B *and* wei $B' > n$.

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