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ORDERS IN SEMILOCAL RINGS[1]

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A semilocal ring S is one such that $S/\text{rad } S$ is semisimple (=a direct sum of simple modules). The first theorem generalizes a theorem of Faith and Utumi [65] to semilocal rings, which had been extended by Robson [67] to artinian rings. (Cf. also Procesi [65].)

(A) THEOREM. *If R has a semilocal right quoring (=quotient ring) $S=D_n$ which is a full $n\times n$ matrix ring, then there exists a set M of $n\times n$ matrix units such that R contains a right order F_n of D_n, and F is a right order in $D=$*centralizer *M.*

The proof depends on the lemma.

(B) LEMMA. *If F_n has semilocal right quoring D_n, where F is subring of D, then for every regular element $t\in F_n$, there exists a regular element $a\in F_n$ such that $x=ta=(x_{ij})$ is represented by a matrix (x_{ij}) with diagonal elements x_{ii} regular elements of F, and the off-diagonal elements x_{ij} are contained in $F\cap$rad D.*

This was proved by Faith-Utumi [65] for semisimple D, and Robson [67] for artinian D. The proof of (A) makes use only of the case D is semisimple, and both (B) and (A) require the rather obvious fact that if R has semilocal right quoring S, then $\overline{R}$ has a semisimple right quoring $\overline{S}=S/\text{rad } S$, where $\overline{R}\approx R/(R\cap\text{rad } S)$ is the image of R under the canonical map $S\to\overline{S}$. Then, $\overline{R}$ is semiprime right Goldie.

(A) has the following application.

(C) THEOREM. *Any maximal $\sim_l$ right order in a semilocal ring $S=D_n$ is isomorphic to the endomorphism ring of a torsion-free unital module over a right order of D.*

The proof is patterned after that of Faith [64] for the case D is a field, and Robson [67] for D artinian. (*Note.* Two right orders R_1 and

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