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THE FUNCTIONAL-DIFFERENTIAL EQUATION $y'(x) = ay(\lambda x) + by(x)$[1]

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ABSTRACT. The paper discusses the functional-differential equation

$$(1) \qquad y'(x) = ay(\lambda x) + by(x) \qquad (0 \leqq x < \infty),$$

where a is a possibly complex constant, b a real constant, and λ a nonnegative constant.

The paper first shows that the boundary-value problem associated with (1) and the boundary condition

$$y(0) = 1$$

is well-posed if $\lambda < 1$, but not if $\lambda > 1$.

The remainder of the paper discusses the asymptotic properties of solutions of the equation as $x \to \infty$. If $\lambda < 1$, it is possible to discuss the asymptotics of *all* solutions of the equation; if $\lambda > 1$, it is shown that, given a specific asymptotic behavior, there is one and only one solution which possesses that asymptotic behavior.

1. Introduction.

The functional-differential equation

$$(1.1) \qquad y'(x) = ay(\lambda x) + by(x) \qquad (0 \leqq x < \infty)$$

arises as a mathematical idealization and simplification of an industrial problem involving wave motion in the overhead supply line to an electrified railway system [1]. (It is curious that the particular case $b = 0$ also appears in the quite different context of a partitioning problem in the theory of numbers [2].) In the problem as it arises in practice, a and b are real constants, and y is a real-valued function, but without significant complication we can (and shall) allow complex values for a while retaining b real. The case $a = 0$ is trivial and we shall always assume $a \neq 0$.

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[1] An expanded version of an invited address delivered by the first author to the 680th meeting of the Society in Pasadena, California on November 21, 1970; received by the editors June 17, 1971.

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