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FINITE SIMPLE GROUPS OF LOW 2-RANK AND THE FAMILIES $G_2(q)$, $D_4^2(q)$, q ODD[1]

BY DANIEL GORENSTEIN AND KOICHIRO HARADA

ABSTRACT. In §1, we describe the presently known finite simple groups of 2-ranks 3 and 4 and discuss in outline the existing methods of characterizing such groups by means of their Sylow 2-subgroups. In the balance of the paper, we illustrate these techniques concretely by classifying all simple groups whose Sylow 2-subgroups are isomorphic to those of $G_2(q)$ for some odd q.

1. Simple groups of low 2-rank. The *2-rank* of a group G is, by definition, the maximum rank of an abelian 2-subgroup of G. Thus a group of 2-rank 0 is of odd order and so is solvable by the Feit-Thompson theorem [59]. A group of 2-rank 1 has either cyclic or generalized quaternion Sylow 2-subgroups and so is not simple (assuming it is of composite order) by Burnside's transfer theorem and a theorem of Brauer and Suzuki [58]. Recently it has been shown by Alperin, Brauer, and Gorenstein [56], using various previously established classification theorems, that the only simple groups of 2-rank 2 are the groups

$$L_2(q),\ L_3(q),\ U_3(q),\quad q \text{ odd},\qquad U_3(4),\ A_7, \text{ and } M_{11}.$$

Work on the determination of simple groups of 2-rank 3 and 4 has only recently begun. As in the case of groups of 2-rank 2, the analysis divides into two major parts:

A. Determination of the possible Sylow 2-subgroups of a simple group of 2-rank 3 or 4.

B. Classification of simple groups having Sylow 2-subgroups of each of the types specified under A.

At the present time, work on problem A is only just beginning. However, whatever the outcome of such an analysis, the list of possible Sylow 2-subgroups will obviously include those of all the presently known simple groups of 2-rank 3 or 4. Thus it will be instructive to list these groups.

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