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PERTURBATION OF PERIODIC MANIFOLDS OF HAMILTONIAN SYSTEMS

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Introduction. It is well known that a closed orbit of a hamiltonian system is preserved under small perturbations of the hamiltonian function, provided that the orbit is nondegenerate in a certain sense [5, §38]. In this announcement, we describe some results about the behavior of manifolds of closed orbits of hamiltonian systems under perturbation of the hamiltonian function. This yields, in turn, new results concerning the existence of closed orbits near equilibria for which the linearized system exhibits degenerate behavior.

1. **Periodic manifolds defined.** Let (M, ω) be a symplectic manifold. (We follow the notation of [1], to which we refer the reader for definitions left unspecified here.) Any real-valued function H on M gives rise to a hamiltonian vector field X_H on M. A submanifold $\Sigma \subseteq M$ is called a *periodic manifold* of X_H if the following conditions are satisfied:

(PM 1). Σ is an invariant manifold of X_H; i.e., X_H is tangent to Σ at all points of Σ.

(PM 2). All the orbits of X_H on Σ are closed, and their periods all divide a number $\tau > 0$, called a period of Σ.

(PM 3). H is constant on Σ.

Let $\{F_t\}_{t\in\mathbb{R}}$ be the flow generated by X_H. If Σ is a periodic manifold with period τ, F_τ is the identity on Σ, and T_mF_τ maps T_mM into itself for each $m\in\Sigma$. If c is the value of H on Σ, T_mF_τ maps $T_m[H^{-1}(c)]$ into itself and leaves the subspace $T_m\Sigma$ fixed. T_mF_τ induces, therefore, a map $T_m^\nu F_\tau$ of the "normal" space, $T_m[H^{-1}(c)]/T_m\Sigma$, into itself. We call Σ *nondegenerate* if $T_m^\nu F_\tau$ does not have 1 as an eigenvalue for any $m\in\Sigma$.

For technical reasons connected with our method of proof, we need to impose a further condition on periodic submanifolds, called regularity. The precise definition is given in [11].

2. **Examples.** 1. If Σ consists of a single closed orbit γ, Σ is nondegenerate if and only if the characteristic multiplier 1 has multi-

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