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THE FINITENESS OF I WHEN $R[X]/I$ IS FLAT

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Let R be a commutative ring with identity, let X be a single indeterminate, and let I be an ideal of $R[X]$. Denote by min I the set $\{f\neq 0\in I \mid \deg f\leqq \deg g \text{ for all } g\neq 0\in I\}$. Let $c(I)$ denote the ideal of R generated by the coefficients of the elements of I. We use $\bar{R}$ for the integral closure of R (in its total quotient ring) and $J(R)$ for the intersection of the maximal ideals of R. By a regular element, we mean a nonzero-divisor. An R-module M is called torsion-free if $rm=0$, $r\in R$, $m\neq 0\in M$, implies r is a zero-divisor of R.

1. **Main results.** (Proofs and details will appear elsewhere.) We assume throughout this section that min I contains a regular element of $R[X]$.

1.1 THEOREM. *If $R[X]/I$ is a flat R-module, then I is a finitely generated ideal of $R[X]$.*

The proof proceeds as follows. First prove the theorem in the case that R is quasi-local integrally closed with infinite residue field. Then remove the infinite residue field assumption by adjoining an indeterminate. Next remove the quasi-local assumption, and finally remove the assumption that R be integrally closed.

If R is integrally closed, the generators of I in 1.1 can be chosen from min I. In proving 1.1, we obtain the following more precise result in the case that R is quasi-local integrally closed.

1.2 THEOREM. *If R is quasi-local integrally closed, then the following are equivalent:*

(i) *I is principal and $c(I)=R$.*

(ii) *$R[X]/I$ is R-flat.*

(iii) *$R[X]/I$ is R-torsion-free and $c(I)=R$.*

Actually, (i)$\Rightarrow$(ii)$\Rightarrow$(iii) is valid for arbitrary R, while only (iii)$\Rightarrow$(i) requires that R be quasi-local integrally closed. ((ii)$\Rightarrow$(i) can also be proved for slightly more general R, namely if R is the integral closure

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