BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 5, September 1971

AN ALGEBRA ASSOCIATED TO A COMBINATORIAL GEOMETRY

BY WILLIAM GRAVES[1]

Communicated by Gian-Carlo Rota, March 17, 1971

1. **Preliminaries.** A functor from a category of combinatorial geometries, or equivalently a category of geometric lattices, to a category of commutative algebras will be described, and some properties of this functor will be investigated. In particular, a cohomology will be associated to each point of a geometry and will be derived from the associated algebra.

If (G, S) is a geometry on a set S [1, p. 2.4], then $L(G)$, or simply L when no opportunity for confusion exists, denotes the associated geometric lattice of closed subsets of S. The rank function of L or G is denoted r.

A morphism

$$\sigma: (G, S) \to (G', S')$$

of geometries is a function

$$\sigma: S \cup \{0\} \to S' \cup \{0\}$$

such that $\sigma(0)=0$ and the inverse image of a closed subset of S' is a closed subset of S. It is precisely the latter condition which is necessary and sufficient [1, p. 9.17] to extend σ to a strong map [1, p. 9.3]

$$\sigma: L(G) \to L(G').$$

The category of geometries with morphisms defined as above is equivalent to the category of geometric lattices and strong maps. These two categories will be used interchangeably throughout.

2. **The functor** $(G, S) \to A(G)$. Let k be a commutative ring (with 1). For any geometry (G, S), let $P(G)$ be the symmetric k-algebra on the free k-module on S; that is, the k-algebra of polynomials in S. Let $J(G)$ be the ideal of $P(G)$ generated by all monomials in S and differences of monomials in S of the forms

AMS 1969 *subject classifications*. Primary 0535.

Key words and phrases. Combinatorial geometry, matroid, geometric lattice, commutative algebra, graded algebra, cohomology.

[1] This research was partially supported by NSF Grant GP-11822.