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# ON THE MEAN CURVATURE OF SUBMANIFOLDS OF EUCLIDEAN SPACE

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Let $x: M^n \to E^m$ be an immersion of an $n$-dimensional manifold $M^n$ in a euclidean space $E^m$ of dimension $m$ $(m > n > 1)$, and let $\nabla$ and $\nabla'$ be the covariant differentiations of $M^n$ and $E^m$, respectively. Let $u$ and $v$ be two tangent vector fields on $M^n$. Then the second fundamental form $h$ is given by

$$\nabla'_u v = \nabla_u v + h(u, v). \tag{1}$$

If $\{e_1, \cdots, e_n\}$ is an orthonormal basis in the tangent space $T_p(M)$ at $p \in M^n$, then the mean curvature vector $H(p)$ at $p$ is given by

$$H(p) = (1/n) \sum_{i=1}^{n} h(e_i, e_i). \tag{2}$$

Let $\langle\ ,\ \rangle$ denote the scalar product of $E^m$. If there exists a function $f$ on $M$ such that $\langle h(u, v), H\rangle = f\langle u, v\rangle$ for all tangent vector fields $u$, $v$ on $M^n$, then $M^n$ is called a *pseudo-umbilical submanifold* of $E^m$. If the covariant derivative of $H$ in $E^m$ is tangent to $x(M^n)$ everywhere, then $H$ is said to be parallel in the normal bundle. In [2], [3], the author proved that if $M^n$ is closed, then the mean curvature vector $H$ satisfies

$$\int_{M^n} \langle H, H\rangle^{n/2} dV \geqq c_n, \tag{3}$$

where $dV$ denotes the volume element of $M^n$ and $c_n$ is the area of the unit $n$-sphere. The equality sign of (3) holds when and only when $M^n$ is imbedded as a hypersphere in an $(n+1)$-dimensional linear subspace of $E^m$. It is interesting to know whether the inequality (3) can be improved for some special submanifolds of $E^m$.

The main purpose of this paper is to announce some results in this direction together with some results on pseudo-umbilical submanifolds. Details will appear elsewhere.

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