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MULTIPLICITY FORMULAS FOR CERTAIN SEMISIMPLE LIE GROUPS

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1. **Introduction.** The main purpose of this note is to announce a result (Theorem 5) concerning finite-dimensional representations of semisimple Lie groups of real rank 1. Theorem 5 extends [**5**, Corollary 3.8], which states that the finite-dimensional spherical representations are the conical ones, and [**7**, Corollary 1 of Theorem 2.1], which asserts the existence of minimal types for finite-dimensional representations of complex groups. Our method, based on a previously unpublished general formula (see §2) due to B. Kostant, yields several other multiplicity results as well.

Let H_1 be a real Lie group and let H_2 be a Lie subgroup of H_1. Let $\alpha \in \hat{H}_1$ (^ denotes the set of equivalence classes of *finite-dimensional* continuous complex irreducible representations), and assume that the restriction to H_2 of any member of α splits into a direct sum of irreducible representations of H_2. For all $\beta \in \hat{H}_2$, let $m(\alpha, \beta)$ denote the corresponding multiplicity.

We are concerned here with the case in which H_1 is a connected real semisimple Lie group G of real rank 1, and H_2 is the connected Lie subgroup K corresponding to $\mathfrak{k}$, where $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ is a Cartan decomposition of the Lie algebra of G. The solution of the problem of computing the multiplicities for the pair (G, K) is contained in the solution of the problem for the "dualized" pair (U_1, U_2). Here U_1 is the simply connected compact Lie group with Lie algebra $\mathfrak{k} + i\mathfrak{p} \subset \mathfrak{g}_C$ (the complexification of $\mathfrak{g}$), and U_2 is the connected compact Lie subgroup of U_1 corresponding to $\mathfrak{k}$.

It is well known (see [**4**, Chapter IX] for the notation and classification) that if the Lie algebra of U_1 is assumed simple, there are five possibilities for the pair (U_1, U_2):

Type A_n: $(SU(n+1), S(U_1 \times U_n))$ (special unitary case)
Type B_n: $(\mathrm{Spin}(2n+1), \mathrm{Spin}(2n))$ (orthogonal case)

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[1] The results announced here are contained in the author's Ph.D. thesis, written under the guidance of Professor B. Kostant and Professor S. Helgason at M.I.T., where the author held an N.S.F. Graduate Fellowship.