

$B_{(\text{TOP}_n)\sim}$ AND THE SURGERY OBSTRUCTION¹

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Communicated by M. F. Atiyah, February 16, 1971

This note announces "calculations" of the homotopy type of $B_{(\text{TOP}_n)\sim}$ and the nonsimply-connected surgery obstruction. Proofs, more precise statements, and consequences will appear in [6].

Remove the extraneous 2-torsion from KO by forming the pullback

$$\begin{array}{ccc} B_0^* & \longrightarrow & \prod_i (K(Z[1/\text{odd}], 4i)) \\ \downarrow & & \downarrow \\ B_0 \otimes Z[\frac{1}{2}] & \xrightarrow{ph} & \prod_i K(Q, 4i), \end{array}$$

and define

$$L = B_0^* \times \prod_i K(Z/2, 4i + 2).$$

L is a periodic multiplicative spectrum with product \otimes in B_0^* , and cohomology multiplication in the $Z/2$ part. B_0^* acts on the $Z/2$ part by reduction mod 2, which gives $\prod_i K(Z/2, 4i)$, and inclusion in $\prod_i K(Z/2, 2i)$.²

Students of surgery will recognize Sullivan's calculation in [7] as $G/\text{TOP} \times Z \simeq L$. The Whitney sum in G/TOP , however, is given by $a \oplus b = a + b + 8a \otimes b$ in L .

THEOREM 1. *Topological block bundles are naturally oriented in L . If $B_{L G_n}$ is the classifying space for L -oriented G_n bundles, this induces a diagram of fibrations, for $n \geq 3$,*

AMS 1970 subject classifications. Primary 57D65, 55F60, 57C50; Secondary 55C05, 57B10, 55B20, 20F25.

Key words and phrases. Surgery, Poincaré duality, topological block bundles.

¹ This work was partially supported by the National Science Foundation grant GP 20307 at the Courant Institute of New York University.

² (ADDED IN PROOF.) This cohomology structure was deduced using product formulas inferred from [7], [8]. This formula is now known to be wrong, and modified versions have been obtained by several groups. A slightly more complicated structure is thus required on L , and will be corrected in [6].