$B_{(\text{TOP}_n)}\sim$ AND THE SURGERY OBSTRUCTION[1]

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This note announces "calculations" of the homotopy type of $B_{(\text{TOP}_n)}\sim$ and the nonsimply-connected surgery obstruction. Proofs, more precise statements, and consequences will appear in [6].

Remove the extraneous 2-torsion from KO by forming the pullback

$$
\begin{array}{ccc}
B_0^* & \longrightarrow & \prod_i \left(K(Z[1/\text{odd}], 4i)\right) \\
\downarrow & & \downarrow \\
B_0 \otimes Z[\tfrac{1}{2}] & \xrightarrow{\;ph\;} & \prod_i K(Q, 4i),
\end{array}
$$

and define

$$
L = B_0^* \times \prod_i K(Z/2, 4i + 2).
$$

L is a periodic multiplicative spectrum with product $\otimes$ in B_0^*, and cohomology multiplication in the $Z/2$ part. B_0^* acts on the $Z/2$ part by reduction mod 2, which gives $\prod_i K(Z/2, 4i)$, and inclusion in $\prod_i K(Z/2, 2i)$.[2]

Students of surgery will recognize Sullivan's calculation in [7] as $G/\text{TOP} \times Z \simeq L$. The Whitney sum in G/TOP, however, is given by $a \oplus b = a + b + 8a \otimes b$ in L.

THEOREM 1. *Topological block bundles are naturally oriented in* L. *If* B_{LG_n} *is the classifying space for* L-*oriented* G_n *bundles, this induces a diagram of fibrations, for* $u \geq 3$,

[2] (ADDED IN PROOF.) This cohomology structure was deduced using product formulas inferred from [7], [8]. This formula is now known to be wrong, and modified versions have been obtained by several groups. A slightly more complicated structure is thus required on L, and will be corrected in [6].